

Solution to Homework 11

Question. If a value is -1 with probability 0.25 , 0 with probability 0.25 , and 1 with probability 0.5 , then how many binary questions do we have to ask, on average, to find the exact value?

Answer. The average number of binary questions is determined by the entropy $S = -\sum_i p_i \cdot \log_2(p_i)$, where p_i are the corresponding probabilities. In our case,

$$\begin{aligned} S &= -0.25 \cdot \log_2(0.25) - 0.25 \cdot \log_2(0.25) - 0.5 \cdot \log_2(0.5) = \\ &= -0.25 \cdot (-2) - 0.25 \cdot (-2) - 0.5 \cdot (-1) = 0.5 + 0.5 + 0.5 = 1.5. \end{aligned}$$