

Solution to Homework 14

Question. Suppose that we have three measurements of the same quantity, resulting in values -1 , 0 , and 1 . Use the Least Squares method to come up with the most probable estimate of the actual value of the measured quantity.

Answer. According to the least squares method, we need to find the value a for which the sum of the squares of the differences between the observations and the value a is the smallest possible. In our case, this means that we minimize the expression

$$(a - (-1))^2 + (a - 0)^2 + (a - 1)^2.$$

Differentiating this expression with respect to the unknown a and equating the resulting derivative to 0, we conclude that

$$2(a - (-1)) + 2(a - 0) + 2(a - 1) = 0.$$

If we divide both sides by a common factor 2, we get

$$a + 1 + a + a - 1 = 0,$$

so $3a = 0$ and the desired estimate is $a = 0$.