Solution to Homework 20

Question. Suppose that $y = f(x_1, x_2) = x_1^2 + x_2^2$. Suppose that:

- with confidence 1, experts believe that the actual value of x_1 is in the interval $[0.8, 1.2]$, and that actual value of x_2 is in the interval $[1.6, 2.4]$;
- with confidence 0.5, experts believe that the actual value of x_1 is in the interval $[0.9, 1.1]$, and that actual value of x_2 is in the interval $[1.8, 2.2]$; and
- with confidence close to 0, experts believe that the actual value of x_1 is 1, and that actual value of x_1 is 2.

Describe the corresponding alpha-cuts for y.

Hint: The above function $y = f(x_1, x_2)$ is strictly increasing with respect to both inputs x_i – when they are both positive. So, when the inputs x_i are located in intervals, this function:

- attains its smallest value when both x_i are the smallest possible, and
- attains its largest value when both x_i are the largest possible.

Answer. In general, confidence c corresponds to the alpha-level $1 - c$. So:

- confidence 1 corresponds to $\alpha = 1 - 1 = 0$;
- confidence 0.5 corresponds to $\alpha = 1 - 0.5 = 0.5$; and
- confidence close to 0 corresponds to $\alpha = 1 - 0 = 1$.

So, for $\alpha = 0$ – which corresponds to confidence 1 – the alpha-cut is:

$$[0.8^2 + 1.6^2, 1.2^2 + 2.4^2] = [0.64 + 2.56, 1.44 + 5.76] = [3.2, 7.2].$$

So, for $\alpha = 0.5$ – which corresponds to confidence 0.5 – the alpha-cut is:

$$[0.9^2 + 1.8^2, 1.1^2 + 2.2^2] = [0.81 + 3.24, 1.21 + 4.84] = [4.05, 6.05].$$

For $\alpha = 1$ – which corresponds to confidence close to 0 – the alpha-cut consists of the single point:

$$1^2 + 2^2 = 5.$$