Solution to Homework 22

Question. If we have three alternatives, with gains $[1,2]$, $[0,3]$, and $[-2,-1]$, which of them are possibly optimal? definitely optimal? Which of the alternatives should we choose if Hurwicz coefficient is α_H is 0.2? 0.5? 0.8?

Answer. An alternative is definitely optimal if its lower endpoint is larger than or equal to all other upper endpoints. One can check that in this case, there is no such alternative:

- for Alternative 1, its lower endpoint 1 is not larger than the upper endpoint 3 of the second alternative;
- for Alternative 2, its lower endpoint 0 is not larger than the upper endpoint 2 of the first alternative; and
- for Alternative 3, its lower endpoint -2 is not larger than any of the other upper endpoints 2 and 3.

An alternative is possibly optimal if its upper endpoint is larger than or equal to all the lower endpoints – i.e., equivalently, to the maximum of the lower endpoints. In our case, this maximum is equal to $\max(1,0,-2)=1$. So, an alternative is possible optimal if its upper endpoint is larger than or equal to 1. This property is true only for Alternatives 1 and 2, so these are the only possible optimal alternatives.

In the Hurwicz approach, we replace each interval $[\underline{x},\overline{x}]$ with the value $x = \alpha_H \cdot \overline{x} + (1-\alpha_H)\cdot \underline{x}$, and select the alternative for which this number if the largest. So:

- For $\alpha_H = 0.2$, we have:

$$x_1 = 0.2 \cdot 2 + (1-0.2)\cdot 1 = 0.2\cdot 2 + 0.8 \cdot 1 = 0.4 + 0.8 = 1.2,$$

$$x_2 = 0.2 \cdot 3 + (1-0.2)\cdot 0 = 0.2\cdot 3 + 0.8 \cdot 0 = 0.6 + 0 = 0.6,$$

$$x_3 = 0.2\cdot(-1)+(1-0.2)\cdot(-2) = 0.2\cdot(-1)+0.8\cdot(-2) = -0.2-1.6 = -1.8,$$

so we select Alternative 1.

- For $\alpha_H = 0.5$, we have:

$$x_1 = 0.5 \cdot 2 + (1-0.5)\cdot 1 = 0.5\cdot 2 + 0.5 \cdot 1 = 1 + 0.5 = 1.5,$$

$$x_2 = 0.5 \cdot 3 + (1-0.5)\cdot 0 = 0.5\cdot 3 + 0.5 \cdot 0 = 1.5 + 0 = 1.5,$$

$x_3 = 0.5 \cdot (-1) + (1 - 0.5) \cdot (-2) = 0.5 \cdot (-1) + 0.5 \cdot (-2) = -0.5 - = -1.5,$

so we both Alternatives 1 and 2 are equally good.

- For $\alpha_H = 0.8$, we have:

$$x_1 = 0.8 \cdot 2 + (1 - 0.8) \cdot 1 = 0.8 \cdot 2 + 0.2 \cdot 1 = 1.6 + 0.2 = 1.8,$$

$$x_2 = 0.8 \cdot 3 + (1 - 0.8) \cdot 0 = 0.8 \cdot 3 + 0.2 \cdot 0 = 2.4 + 0 = 2.4,$$

$x_3 = 0.8 \cdot (-1) + (1 - 0.8) \cdot (-2) = 0.8 \cdot (-1) + 0.2 \cdot (-2) = -0.8 - 0.4 = -1.2,$

so we select Alternative 2.