Solution to Homework 23

Question. In the classical logic, implication $A \to B$ is defined as $B \vee \neg A$. Similarly to what we did in class, use linear interpolation to come up with the fuzzy value for $f_\to(0.7, 0.8)$.

Answer. When all the truth values are either true (1) or false (0), we have:

$$f_\to(0,0) = 0 \vee \neg 0 = 0 \vee 1 = 1;$$

$$f_\to(0,1) = 1 \vee \neg 0 = 1 \vee 1 = 1;$$

$$f_\to(1,0) = 0 \vee \neg 1 = 0 \vee 0 = 0;$$

$$f_\to(0,0) = 1 \vee \neg 1 = 1 \vee 0 = 1.$$

The general formula for linear interpolation has the following form: if we know the values $f(x_1)$ and $f(x_2)$, then for every other x, we have

$$f(x) = \frac{x_2 - x}{x_2 - x_1} \cdot f(x_1) + \frac{x - x_1}{x_2 - x_1} \cdot f(x_2).$$

To get the value $f_\to(0.7, 0.8)$, we can apply linear interpolation by x_2 to the values $f_\to(0.7, 0)$ and $f_\to(0.7, 1)$:

$$f_\to(0.7, 0.8) = \frac{1 - 0.8}{1 - 0} \cdot f_\to(0.7, 0) + \frac{0.8 - 0}{1 - 0} \cdot f_\to(0.7, 1) =$$

$$0.2 \cdot f_\to(0.7, 0) + 0.8 \cdot f_\to(0.7, 1).$$

To find the values $f_\to(0.7, 0)$ and $f_\to(0.7, 1)$, we can apply interpolation over x_1:

- To find $f_\to(0.7, 0)$, we can apply interpolation over x_1 to the known values $f_\to(0,0) = 1$ and $f_\to(1,0) = 0$:

$$f_\to(0.7, 0) = \frac{1 - 0.7}{1 - 0} \cdot f_\to(0,0) + \frac{0.7 - 0}{1 - 0} \cdot f_\to(1,0) =$$

$$0.3 \cdot 1 + 0.7 \cdot 0 = 0.3.$$

- To find $f_\to(0.7, 1)$, we can apply interpolation over x_1 to the known values $f_\to(0,1) = 1$ and $f_\to(1,1) = 1$:

$$f_\to(0.7, 0) = \frac{1 - 0.7}{1 - 0} \cdot f_\to(0,1) + \frac{0.7 - 0}{1 - 0} \cdot f_\to(1,1) =$$

$$0.3 \cdot 1 + 0.7 \cdot 1 = 0.3 + 0.7 = 1.$$

Thus:

$$f_{\to}(0.7, 0.8) = 0.2 \cdot f_{\to}(0.7, 0) + 0.8 \cdot f_{\to}(0.7, 1) =$$

$$0.2 \cdot 0.3 + 0.8 \cdot 0.06 + 0.8 = 0.86.$$