Test 2 for Uncertainty in AI class
Fall 2025

1. Suppose that we know the measurement results $\widetilde{x}_1, \ldots, \widetilde{x}_n$, the data processing algorithm $y = f(x_1, \ldots, x_n)$, and the standard deviations $\sigma_1, \ldots, \sigma_n$. Describe how to best estimate the standard deviation σ of the data processing result y when n is small (provide the formula) and how to estimate it when n is large (just explain the idea).

2. Suppose that we know the measurement results $\widetilde{x}_1, \ldots, \widetilde{x}_n$, the data processing algorithm $y = f(x_1, \ldots, x_n)$, and the upper bounds $\Delta_1, \ldots, \Delta_n$ on the absolute values of the measurement errors. Describe how to best estimate the upper bound Δ of the absolute value of the approximation error of the measurement result y when n is small (provide the formula) and how to estimate it when n is large (just explain the idea).

3. Suppose that we use bisection to compute $\sqrt{2}$, i.e., the solution to the equation $f(x) = 0$ when $f(x) = x^2 - 2$. We know that $f(1) = 1^2 - 2 = 1 - 2 = -1 < 0$ and that $f(3) = 3^2 - 2 = 9 - 2 = 7 > 0$, so we know that the solution is somewhere on the interval $[1, 3]$. When we follow bisetion method, what would we do next, and what will be the resulting new narrower interval?

4. Suppose that $y = f(x_1, x_2) = x_1^2 - x_2^2$. Suppose that with confidence 0.5, experts believe that the actual value of x_1 is in the interval $[1, 2]$, and that actual value of x_2 is in the interval $[2, 3]$. Describe the corresponding alpha-cut for y.

Hint: The above function $y = f(x_1, x_2)$ is strictly increasing with respect to x_i and strictly decreasing with respect to x_2 – when both x_i are positive. So, when the inputs x_i are located in intervals, this function:

- attains its smallest value when x_1 is the smallest possible and x_2 is the largest possible, and
- attains its largest value when x_1 is the largest possible and x_2 is the smallest possible.

5. Suppose that our degree of confidence in a statement A is 0.8, in a statement B is 0.7, and in a statement C is 0.6. Suppose that we use min as "and" and max as "or". What is our estimate for the degree of confidence in a composite statement $A \vee (\neg B \,\&\, C)$?

6. If we have two alternatives, with gains $[2, 3]$ and $[1, 4]$, which of them are possibly optimal? definitely optimal? Which of the alternatives should we choose if Hurwicz coefficient α_H is 0.4?

7. Prove that for each final invariant optimality criterion, the optimal alternative is itself invariant.

8. In the classical logic, the equivalence $A \equiv B$ is defined as $f_\equiv(0,0) = f_\equiv(1,1) = 1$ and $f_\equiv(0,1) = f_\equiv(1,0) = 0$. Use linear interpolation to come up with a formula for $f_\equiv(a, b)$, and use this formula to come up with the fuzzy value $f_\equiv(0.7, 0.8)$. Why do we use linear interpolation?

9. Explain, in all necessary details, how to use invariance to explain why ReLU is the optimal activation function. You can use the result that optimal alternative is invariant.