Homework 4

CS 5315 (Theory of Computation)

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1. Describe DIV as a μ-recursive function.

Let $p = m \,\text{DIV}\, n$.

It is known that $p \cdot n + r = m$, where r stands for the remainder of the division.

For $r = 0$, $p \cdot n = m$; and for $r > 0$, $p \cdot n < m$. Therefore, $p \cdot n \le m$.

To express DIV using μ-recursion, we need to find a relationship among p, m and n such that p has the smallest value that will satisfy the DIV properties.

Therefore, DIV can be described by the following μ-recursive function:

$$m \,\text{DIV}\, n = \mu p[(p+1) \cdot n > m] \tag{1}$$

2. Show that the following function $f(n)$ is mu-recursive: $f(n) = 5$ for $n = 3$, $f(n) = 7$ for $n = 5$, and $f(n)$ is undefined for all other n.

Solution: $f(n) = \mu m.[(n = 3 \,\&\, m = 5) \vee (n = 5 \,\&\, m = 7)]$.

3. Describe a Turing Machine that computes the function $f(n) = n + 2$.

(start, #)	$\longrightarrow$	(go_right, R)
(go_right, 1)	$\longrightarrow$	(go_right, R)
(go_right, #)	$\longrightarrow$	(add_one, 1)
(add_one, 1)	$\longrightarrow$	(go_right_again, R)
(go_right_again, #)	$\longrightarrow$	(add_two, 1)
(go_right_again, 1)	$\longrightarrow$	(go_right_again, R)
(add_two, 1)	$\longrightarrow$	(back, L)
(back, 1)	$\longrightarrow$	(back, L)
(back, #)	$\longrightarrow$	**halt**