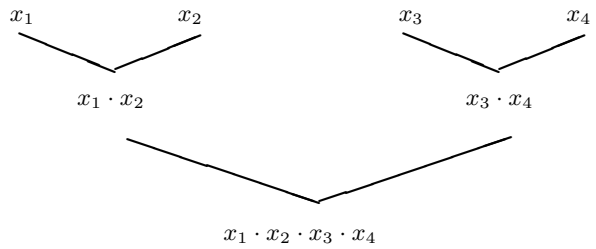
Homework 14
CS 5315 (Theory of Computation)

1. Explain step by step how to compute the product of 4 numbers in parallel.

We want to compute the following product: $x_1 \cdot x_2 \cdot x_3 \cdot x_4$ in parallel. For that, compute $x_1 \cdot x_2$ on one processor P_1, and $x_3 \cdot x_4$ on a different processor P_2. Then, take the results of these computations and multiply them in either one of the processors.

This computation is depicted in the following picture:



2. Product of 4×4 matrices in parallel. Show for just one element.

How many processors, how much time does it takes to calculate?

Given matrices A and B, as shown below:

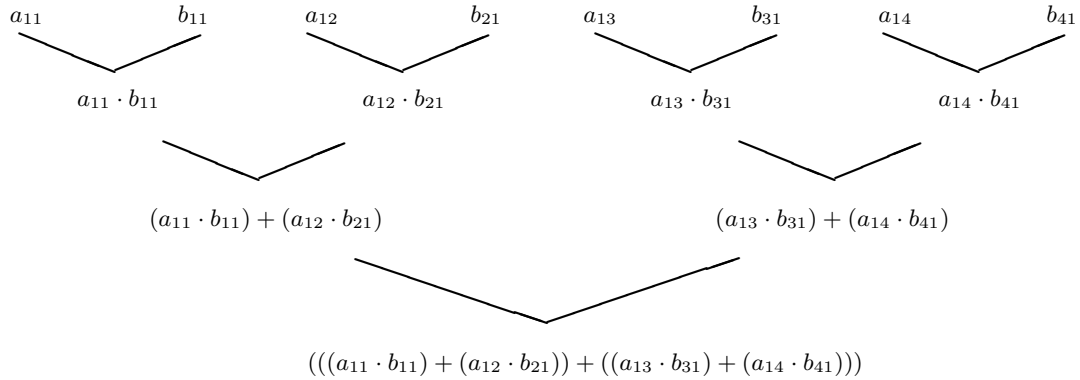
$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \tag{1}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \tag{2}$$

To compute each element c_{ij} of the product of two 4×4 matrices, A and B, in parallel, assign processors to compute the product of a row in matrix A by a column in matrix B, as follows: multiply each of the 4 elements of row of matrix A by each of the 4 elements of column in matrix B, in parallel, and then add the results of each product to get a single value for the product of row i by column j.

As an example, let's compute $c_{11} = a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} + a_{14} \cdot b_{41}$ in parallel.

The following figure depicts the multiplication of the first row of the matrix A by the first column of the matrix B:



For the multiplication of $m \times m$ matrices, m^2 groups of processors will be used to compute each element c_{ij} of the resulting matrix, where $1 \leq i, j \leq m$.

Therefore, for $m = 4$, there will be 16 groups of 4 processors.
Total number of processors is given by $m^3 = 4^3 = 64$.

Not considering communication time, the operations involved are:

32 pairs of numbers multiplied in parallel	(1st step)
16 pairs of numbers multiplied in parallel	(2nd step)
8 pairs of numbers multiplied in parallel	(3rd step)

Total time will be the sum of time necessary for 1 multiplication and 2 additions.

$Time = \log_2 m + 1 = \log_2 4 + 1 = 2 + 1 = 3$ steps.