(state of computer can be described by)

$S_{i,t}$ — state of i-th cell at t-th moment.

$S_{i,b,t}$ — b-th bit in i-th cell at t-th moment.

Tuesday 7th April

$C(x,y)$ — comp. device

P_ℓ — i/p: x, y

Given: x — o/p: T or F

Find y s.t. $C(x,y)$ & $len(y) \le P_\ell(len(x))$

Δt — time quantum

ΔV — smallest size of the cell (A cell can be any part of computer)

S — Largest # of states in a cell

S_{it} — state of cell i at moment t

B — # of bits of to describe an individual state.

$$2^B \geq S = 2^{\log_2 S}$$

$$B \geq \log_2 S$$

$$B = \lceil \log_2 S \rceil \to \text{smallest int} > \log_2 S$$

Example:

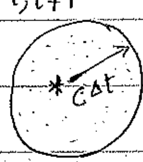
$$\begin{bmatrix} 1\,0\,1 = S_{i,t} \\ 0\,0\,0 = S_{j,t} \\ 1\,0\,1 = S_{i,t+1} \end{bmatrix}$$

($S_{i,2,t}$, $S_{i,1,t}$)

S_{it} consists of bit

& S_{ibt} be the b-th bit in the binary description of the state S_{it}.

$S_{i,t+1}$ depends on the state of different cells is prev moments of time.



distance traversed in $(t+1 - t = \Delta t)$ with speed of light (c) max speed.

$= c\Delta t$

$$N_{neigh} = \text{Neighbour of cells} \le \frac{V}{\Delta V} = \frac{\frac{4}{3}\pi c^3 \Delta t^3}{\Delta V}$$

$$S_{i,t+1} = f_{i,t}(\underbrace{S_{i,t}, S_{jit}}_{\le N_{neigh}}) \to \text{so this means depends on bits describing states}$$

$$S_{i,b,t+1} = f_{i,b,t}(S_{i,1,t}, S_{i,2,t}, \ldots, S_{i,B,t}, \ldots, S_{j,1,t}, \ldots S_{j,B,t})$$

CNF form:—

$\left.\begin{matrix} x_1, \ldots x_n \\ \bar{x}_1, \ldots, \bar{x}_n \end{matrix}\right\}$ literals

$(x_1 \vee x_2) \mathbin{\&} (x_1 \vee x_3)$ — & = AND

CNF form.

By truth table, we can convert to CNF form.

$f(x_1, x_2) \equiv (x_1 = x_2)$.

x_1	x_2	f
F	F	T
F	T	F
T	F	F
T	T	T

$f \equiv (\bar{x}_1 \,\&\, \bar{x}_2) \vee (x_2 \,\&\, x_1)$

$\neg f \equiv (\bar{x}_1 \,\&\, x_2) \vee (x_1 \,\&\, \bar{x}_2)$ } DNF.

De Morgan's laws: $\neg(a \,\&\, b) \equiv \neg a \vee \neg b$

$\neg(a \vee b) \equiv \neg a \,\&\, \neg b$.

$$f \equiv \neg[\neg f] \equiv \neg\left[(\bar{x}_1 \,\&\, x_2) \vee (x_1 \,\&\, \bar{x}_2)\right]$$

$$\equiv \neg(\bar{x}_1 \,\&\, x_2) \wedge \neg(x_1 \,\&\, \bar{x}_2)$$

$$\equiv (x_1 \vee \bar{x}_2) \,\&\, (\bar{x}_1 \vee x_2) \quad \text{CNF}.$$

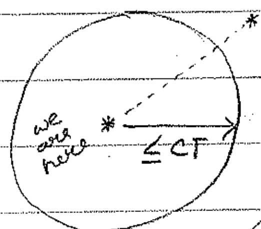
$F_{1,1,1} \,\&\, F_{1,2,1} \,\&\, \dots \,\&\, F_{N,B,T} \,\&\, \dots$

formula showing transition from

1st bit at 1st moment at first cell.

computational time of $C(x,y)$ — another polynomial.

$$T \leq P_c(\text{len}(x) + \text{len}(y)) \leq P_2(n).$$



Now, we know $N \leq \frac{V}{\Delta v} = \frac{4/3\, \pi c^3 (T^3)}{\Delta v}$

$\therefore N \leq Pol^3(n)$

it will reduce to S or !S based on

$(S_{s_1, b_1, 1} = x_1) \,\&\, \dots \,\&\, S_{c_r, b_r, T} = 1$

$S=1 \equiv S$

$S=0 \equiv\, !S$

input is x_1] we are finding here for which x, $C_{x,y} = 1$

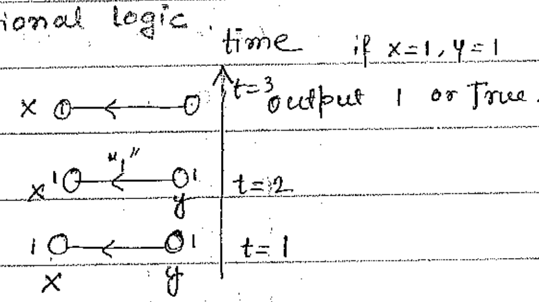
So, we can find value of x, for which $C_{x,y} = 1$, if we can solve this in polynomial time.

So, we have reduced to this propositional logic.

Example:- x – 1 bit

y – 1 bit

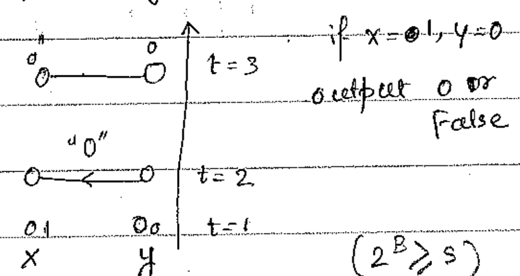
$C(x,y) \equiv (x = y)$ equals



So, here we have 3 cells

x, y, the wire carrying wire

$N = 3$



states x can have $= 2$

y " " $= 2$.

wire " " $= 3$ states.

(Nosignal, 0, 1)

$(2^B \geq S)$

$\therefore$ Max no of state $= 3$ $\quad\therefore$ no of bits to represent 3 state $= 2$

Represent states with bits

(see picture)

cell x : 00 01

cell y : 00 01

wire : no signal 00
"signal 0" 10
"signal 1" 11

For cell x. $S_{112} = S_{111} = 0$ (1: no of bit, 2: moment of time) f_{111}

$S_{122} = S_{121}$ f_{121}

For cell y. $S_{212} = 0$

$S_{222} = S_{221}$

For wire. $S_{312} = 1$

$S_{322} = S_{221}$. f_{322}

f_{111} & f_{121} -- & f_{322}

$= \neg S_{112}$ & $(\bar{S}_{122} \vee S_{121})$ & $(S_{122} \vee \bar{S}_{121})$ & $\neg S_{212}$ & $(\bar{S}_{222} \vee S_{221})$ & $(\bar{S}_{221}$ & $S_{222})$ & $\bar{S}_{312}$ & $(\bar{S}_{322} \vee S_{221}) \vee$
$(S_{322} \vee \bar{S}_{221}) \vee \neg S_{213} \vee$
$(S_{223} \vee \bar{S}_{222})$ & $(\bar{S}_{223} \vee S_{222})$ & $\bar{S}_{313}$ & $\bar{S}_{323}$ & $\bar{S}_{113}$ & $(S_{123} \vee S_{322} \vee S_{122})$
& .... & S_{121} & S_{123} = result is true.

(S_{121} → first state; S_{123} → resulting state.)

For 3rd state. (x)

* $S_{213} = 0$

$S_{223} = S_{222}$.

$S_{313} = 0$

$S_{323} = 0$

$S_{113} = 0$

$$S_{123} = \begin{cases} 1 & \text{if } S_{322} = S_{122} \\ 0 & \text{otherwise.} \end{cases}$$

$$a = \begin{cases} 1 & \text{if } b = c \\ 0 & \text{otherwise} \end{cases}$$

$b = c$ & $a = 1$ then true.

$b \neq c$ & $a = 0$ then true.

a	b	c	f	$\bar{f}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0
0	1	1	0	1

$\bar{f} = [(\bar{a}$ & $\bar{b}$ & $\bar{c}) \vee (\bar{a}$ & b & $c) \vee (a$ & $\bar{b}$ & $c) \vee (a$ & b & $\bar{c})]$

$f = (a \vee b \vee c) \wedge (a \vee \bar{b} \vee \bar{c}) \wedge$
* $(\bar{a} \vee b \vee \bar{c}) \wedge (\bar{a} \vee \bar{b} \vee c)$

H.W. checking if $x \to y$
↓
$\neg x \vee y$.