CNF = conjunctions of
disjunction of literals ← Not $(X_1 \vee \overline{X}_2)$

+ Convert to Skolem

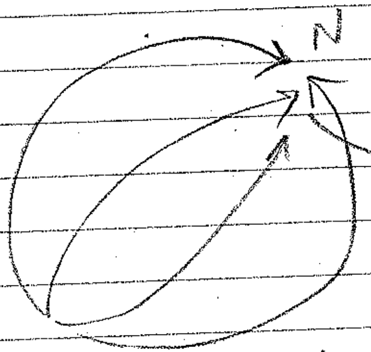
- change name of bound vars : !!

$\forall x A \to C \iff \exists x (A \to C)$
$\exists x A \to C \iff \forall x (A \to C)$

A : it rains a day x $\qquad$ $\forall x$ rainyday(x) $\to$ stay home(y)
C : I stay home
$\iff \exists x$ (rainy-day(x) $\to$ stay(y))

$\exists x$ rainyday(x) $\to$ C
$\iff$ If there exists a rainy day x then I'll stay home

April 9 →



Example 1: 3 CNF formula

+ CNF : C_1 & C_2 & C_3 ...
C_i = $a \vee b \vee c \vee$ ... } NP hard
x_i or $\overline{x_i}$

+ 3-CNF :
$C_i \leq 3$ literals
$a \vee b \vee c$

$(\overline{X_1} \vee X_2 \vee X_3 \vee X_4) \wedge (\overline{X_1} \vee \overline{X_2} \vee \ldots X_3) \wedge \ldots$

Trick : how will the computer do it?

$r_1 = X_1 \vee X_2$
$r_2 = r_1 \vee X_3$
$r_3 = r_3 \vee X_4$
...
...

" $a = b \vee c$ "

a	b	c	f	$\neg f$
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0
0	1	1	0	1

DNF of $\neg f$:

$$(\bar a \wedge \bar b \wedge \bar c) \vee (a \wedge \bar b \wedge c) \vee (a \wedge \bar b \wedge \bar c) \vee$$
$$(\bar a \wedge b \wedge c)$$

$\Leftrightarrow$ CNF of f:

$$f = (a \vee b \vee c) \wedge (\bar a \vee b \vee c) \wedge (\bar a \vee b \vee c) \wedge (a \vee \bar b \vee \bar c).$$

r_1

$\Leftrightarrow \quad \cancel{(r_1 \vee x_1 \vee \bar x_2)}$

$+ \; \overset{a}{r_1} = \overset{b}{x_1} \vee \overset{c}{x_2}$

$\Leftrightarrow \quad (r_1 \vee x_1 \vee x_2) \wedge (r_1 \vee \bar x_1 \vee x_2) \wedge (r_1 \vee \bar x_1 \vee \bar x_2)$
$\wedge (\bar r_1 \vee x_1 \vee x_2) \wedge$

$+ \; \overset{a}{r_2} = \overset{b}{\cancel{x_1}\, r_1} \vee \overset{c}{x_3}$

$$(r_2 \vee r_1 \; x_3) \wedge (r_2 \vee \bar r_1 \vee x_3) \wedge (r_2 \vee \bar r_1 \vee \bar x_3)$$
$$\wedge (\bar r_2 \vee r_1 \vee x_3)$$

r_3

r_2

r_1

$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6$

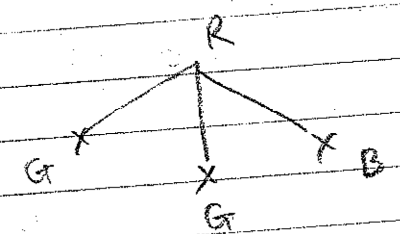
$r_1 \vee x_3 \vee x_4 \vee \dots$

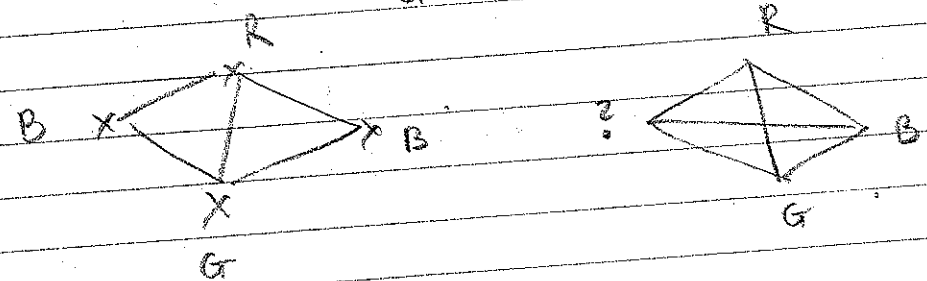
$r_2 \vee x_4 \dots$

$\underbrace{r_3 \vee x_5 \vee x_6}$

3-CNF we stop.

4 coloring :

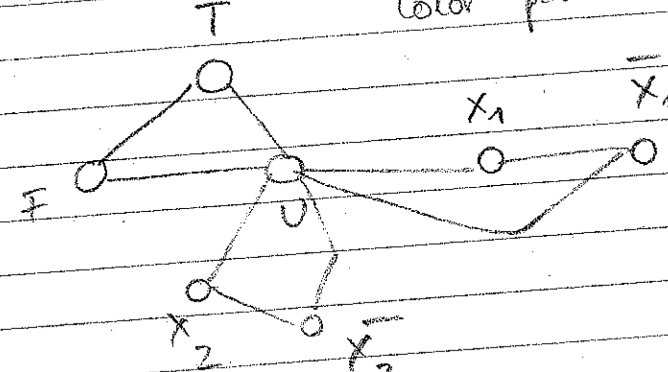




3 - coloring is NP-hard

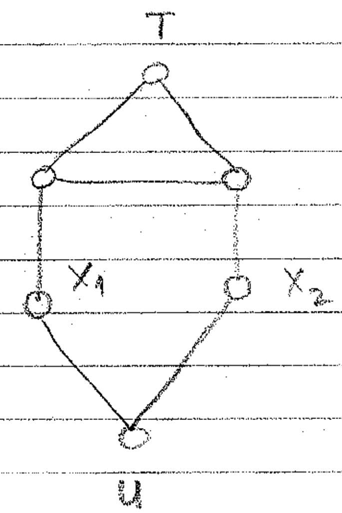
Reduce 3-SAT → 3-color

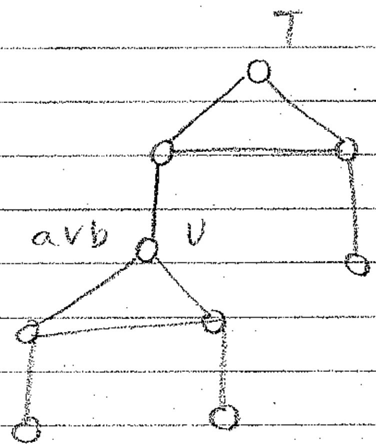
Color palette.



$(a \vee b) \& (a \vee b \vee c) \& \dots$

OR- gadget



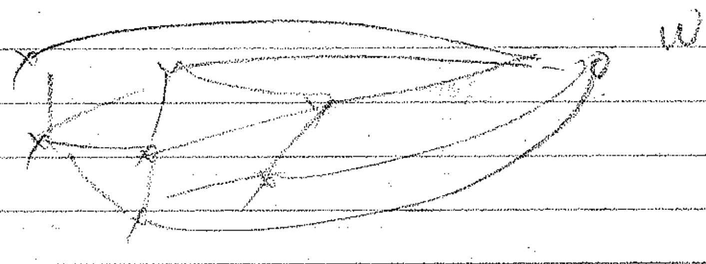


$(x_1 \vee \bar{x}_2) \wedge (x_1 \vee x_2 \vee \bar{x}_3)$

1) Take any form in CNF longer than 3 ⟶ make 3
2) Transform any 3 CNF ⟶ 3-coloring

3) coloring in 5 colors

T F U W



x Subset Sum

$s_1, \dots, s_n$

S - sum

Find a subset $I \subseteq \{1, \dots n\}$

$$\sum_{i \in I} s_i = S$$

x_1	x_2	x_3	x_4	x_5
1	1	5	10	25

$x_1, x_2 \dots x_n \in \{0, 1\}$.

April 14

MODAL LOGIC

+ possibility, necessity, probability

- Main op-s
  - $\Box$ = necessary
  - $\Diamond$ = possible
- Other ops:
  + It was once the case that
  + It will once be the case that
  + It ought to be the case that

$$\Box P \leftrightarrow \neg \Diamond \neg P$$

$$\Diamond P \leftrightarrow \neg \Box \neg P$$

$(\Diamond)$ (P)

→ It's possible that it'll rain.

↔ It's not necessary that it'll not rain.

$\neg\ (\Box)$ $\neg\ (P)$

$$f = (a \vee b \vee c) \,\&\, (a \vee \bar{b} \vee \bar{c}) \,\&\, (\bar{a} \vee b \vee \bar{c}) \,\&\, (\bar{a} \vee \bar{b} \vee c).$$

 : Extra credit

check if $X \to Y \iff Y \vee \neg X$

(1)

$$f = (3.5\, x_1 - 2.6\, x_2 \geq 0)$$

x_1	x_2	f	$\neg f$
0	0	1	0
0	1	0	1
1	0	1	0
1	1	1	0

$$f = (\bar{x}_1 \wedge \bar{x}_2) \vee (x_1 \wedge \bar{x}_2) \vee (x_1 \wedge x_2)$$

$\neg f = \bar{x}_1 \wedge x_2 \longrightarrow$ DNF form for $\neg f$

pue 14 + Jitin: $a \vee b \vee c \vee d$

In class:

$$\underbrace{a \vee b}_{r_1} \vee c \vee d$$

+ Alternative idea:

$$(a \vee b \vee p) \,\&\, (\bar{p} \vee c \vee d)$$

+ Resolution

$$(a \vee b \vee p) \wedge (\bar{p} \vee c \vee d)$$
$$\iff (a \vee b) \vee (c \vee d)$$

NP - hard $\equiv$ can be reduced to the class NP

NP - complete $\equiv$ NP - hard and belongs to class NP.

+ Subset Sum : Given $S_i, \dots S_n, S$

Find $x_i \in \{0, 1\}$ s.t $\sum_{i=1}^{n} X_i S_i = S$

$$F = \underbrace{(X_1 \vee X_2 \vee \bar{X}_3)}_{C_1} \wedge \underbrace{(\bar{X}_1 \vee X_2 \vee X_3)}_{C_2} \wedge \underbrace{(\bar{X}_1 \vee \bar{X}_2)}_{C_3}$$

		X_1	X_2	X_3	C_1	C_2	C_3	$S_{i\to n}$
corr. to X_1	y_1	1	0	0	1	0	0	100, 100
$\bar{X}_1$	z_1	1	0	0	0	1	1	100, 011
x_2	y_2		1	0	1	1	0	10, 110
$\bar{X}_2$	z_2		1	0	0	0	1	10, 001
X_3	y_3			1	0	1	0	1, 010
$\bar{X}_3$	z_3			1	1	0	0	1, 100
Auxiliary	g_1				1	0	0	100
	h_1				1	0	0	100
	g_2					1	0	10
	h_2					1	0	10
	g_3						1	1
	h_3						1	1
$\rightarrow S$		1	1	1	3	3	3	

① if F is satisfiable $\longrightarrow$ we can find X_i s.t $\sum x_i S_i = S$

② If there is a combination s.t. $\sum x_i S_i = S$ then f is satisfiable.

we pick $X_1, X_3, \bar{X}_2$ to satisfy F.

$(X_1 \vee X_2 \vee X_3) \wedge (\bar{X}_1 \vee X_2 \vee X_4) \wedge (\bar{X}_1 \vee \bar{X}_4)$

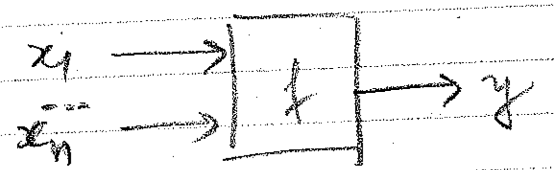
→ No. of auxiliary = # clauses c_1, c_2, c_3
→

Ex of NP-hard :

Given : $f(x_1, \ldots, x_n)$

$[\underline{x}_1, \bar{x}_1], \ldots, [\underline{x}_n, \bar{x}_n]$

Find : $[\underline{y}, \bar{y}] = \{ f(x_1, \ldots x_n);\ x_1 \in [\underline{x}_1, \bar{x}_1], \ldots$
$x_n \in [\underline{x}_n, \bar{x}_n] \}$.



$X_1 \vee \bar{X}_2$

$(X_1 \vee \bar{X}_2)$ $\quad y = (z_1 + (1 - z_2) - z_1(1 - z_2)) \times$
$(\ldots)$

$X_i \in \{0, 1\}$

$y = 1 \iff F$ is satisfiable

$z_1 = [0, 1]$

$\neg z_2 = 1 - z_2$

$X \vee Y = X + Y - X \cdot Y$

$X \wedge Y = X \cdot Y$