Thursday 9th April.

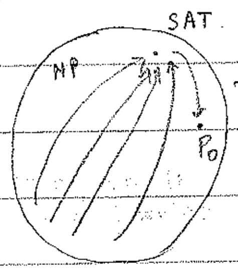
Fuzzy Logic.

$[0,1]$ $\rightarrow$ output lies in this

Quiz on CNF, DNF form

use general algorithm to find CNF & DNF

1. $x_1, x_2 \in \{0,1\}$, $f \equiv$ " $3.5x_1 - 2.6x_2 \geq 0$ "
2. " if $(x_1 \geq x_2)$ then " $(x_1 = x_2)$ "



If SAT can be reduced to P_0, then P_0 is NP-hard.

Example: 3-CNF formula

CNF C_1 & C_2 & ... & C_m } NP-hard

$C_i = a \vee b \vee .. \vee c$, where each literal is x_i or $\bar{x}_i$

3-CNF is same as CNF, but

$C_i \leq 3$ literals

Now, how we reduce CNF to 3-CNF form? Say, CNF =

$(x_1 \vee x_2 \vee \bar{x}_3 \vee x_4)$ & $(\bar{x}_1 \vee x_2 \vee x_3 \vee \bar{x}_4)$ & ...

Trick: how will the computer do it?

The eq is satisfied if

$(r_1 = x_1 \vee x_2)$ &

$(r_2 = r_1 \vee \bar{x}_3)$ &

$(r_2 \vee x_4)$ & are true.

Truth table for " $a = b \vee c$ "

a	b	c	f	¬f
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

CNF $\neg f \equiv (\bar{a} \& \bar{b} \& c) \vee (\bar{a} \& b \& \bar{c}) \vee (\bar{a} \& b \& c) \vee (a \& \bar{b} \& \bar{c})$

$f \equiv (a \vee b \vee \bar{c}) \& (a \vee \bar{b} \vee c) \& (a \vee \bar{b} \vee \bar{c}) \& (\bar{a} \vee b \vee c)$

$(\overset{a}{r_1} = \overset{b}{x_1} \vee \overset{c}{x_2})$ & $(\overset{a}{r_2} = \overset{b}{r_1} \vee \overset{c}{\bar{x}_3})$ & $(r_2 \vee x_4)$

* Replaced by clauses, $(r_1 \vee x_1 \vee \bar{x}_2)$ & $(r_1 \vee \bar{x}_1 \vee x_2)$ & $(r_1 \vee \bar{x}_1 \vee \bar{x}_2)$ & $(\bar{r}_1 \vee x_1 \vee x_2)$ &

$(r_2 \vee r_1 \vee x_3)$ & $(r_2 \vee \bar{r}_1 \vee \bar{x}_3)$ & $(r_2 \vee \bar{r}_1 \vee x_3)$ & $(\bar{r}_2 \vee r_1 \vee \bar{x}_3)$ &

& $(r_2 \vee x_4)$

Thus reduced to 3-CNF

$(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6)$ can be broken into.

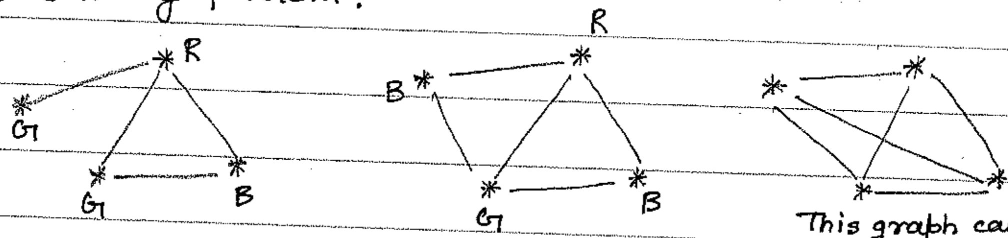
$(r_1 = x_1 \vee x_2)$ &

$(r_2 = r_1 \vee x_3)$ &

$(r_3 = r_2 \vee x_4)$ &

$(r_3 \vee x_5 \vee x_6)$

3-Coloring problem :-



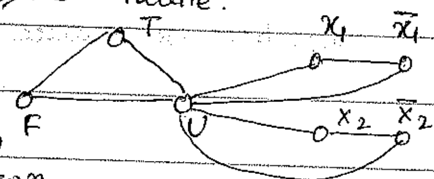
This graph cann't be colored with 3 colors.

3-coloring problem is NP-hard :-

We know 3-CNF is NP-hard. So, we will reduce it 3-coloring problem.

3 colors are – True
False
Unknown.

~~Palette~~ Palette.

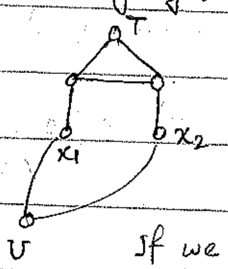


To make sure x_1, $\overline{x_1}$ is not unknown make connection with unknown.

This graph can be colored with 3-colors.

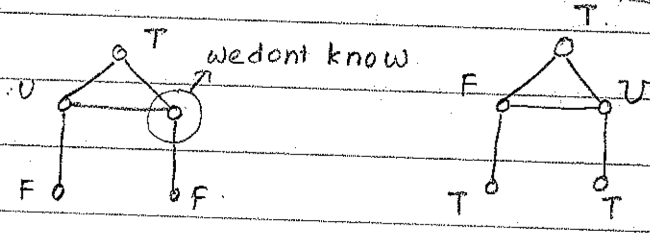
Changing to. $(a \vee b)$ & $(a \vee b \vee c)$ & ...

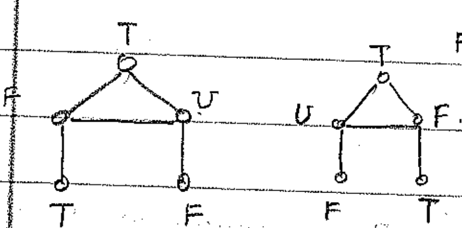
OR-gadget.



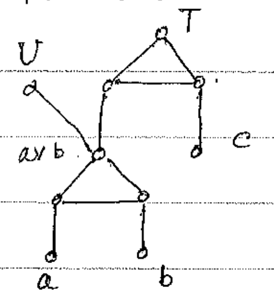
we are sure x_1, x_2 are not U since connected

If we have.



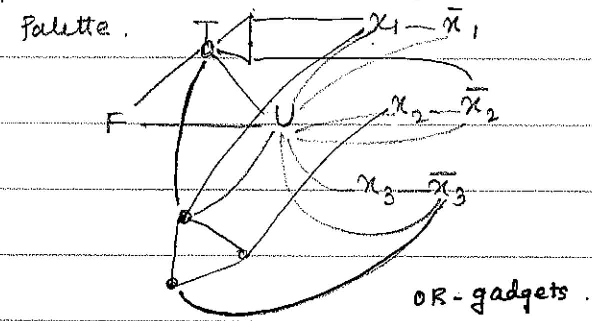


For 3 literals, OR-gadget.



we should avoid having $a \vee b = F$
$c = F$

Example. $(x_1 \vee \bar{x}_2)$ & $(x_1 \vee x_2 \vee \bar{x}_3)$



H.W.

1. Give an example of CNF with > 3 reduce to 3-CNF.
2. Take 3-CNF formula and reduce to 3-coloring.
3. Coloring in 5-colors

Say 4 colors: T F U W

$$\begin{matrix} T \\ F \\ U \end{matrix} \to W$$

If we connect with W, then reduced to coloring with T, F, U.

<u>Subset_Sum</u> :-

$s_1, \dots, s_n$ — integer no $s_i > 0$

S — Sum

Find a Subset $I \subseteq \{1, \dots, n\}$ s.t. $\sum_{i \in I} s_i = S$

$x_1 \, x_2 \dots x_n \in \{0,1\}$

$S = \sum_{i=1}^{n} x_i s_i$

$x_1 = 1 \quad x_2 = 1 \quad x_3 = 1 \quad x_4 = 1 \quad x_5 = 0$

$1 \quad 1 \quad 5 \quad 10 \quad 25.$

$17 = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 5 + 1 \cdot 10$