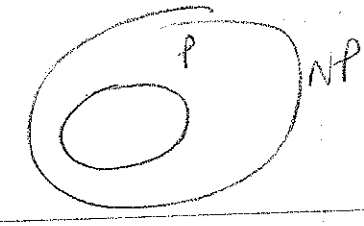


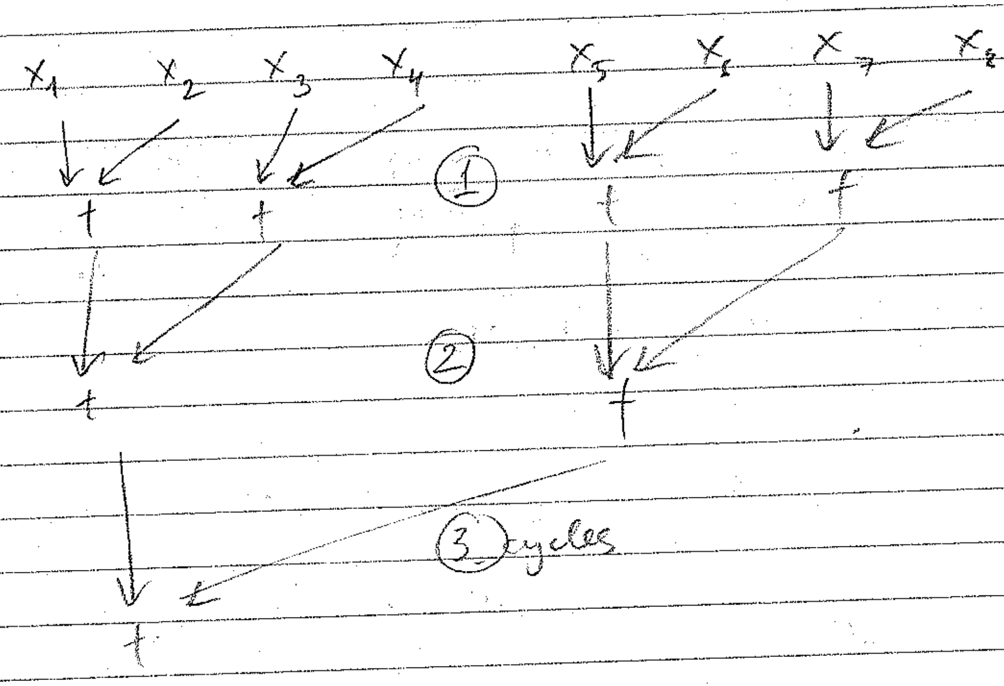
ril 21 — Problem: Polynomial time may still mean too long

+ Solution: distribution, parallelize,

$x_1 + x_2 \dots + x_n$.

If we have 2 processors:

$$\underbrace{(x_1 + x_2 + \dots + x_{n/2})}_{P_1} + \underbrace{(x_{n/2} + \dots + x_n)}_{P_2}$$



$n = 4 \quad t = 2 \quad p = 2$

...

$n = 8 \quad t = 3 \quad p = 4$

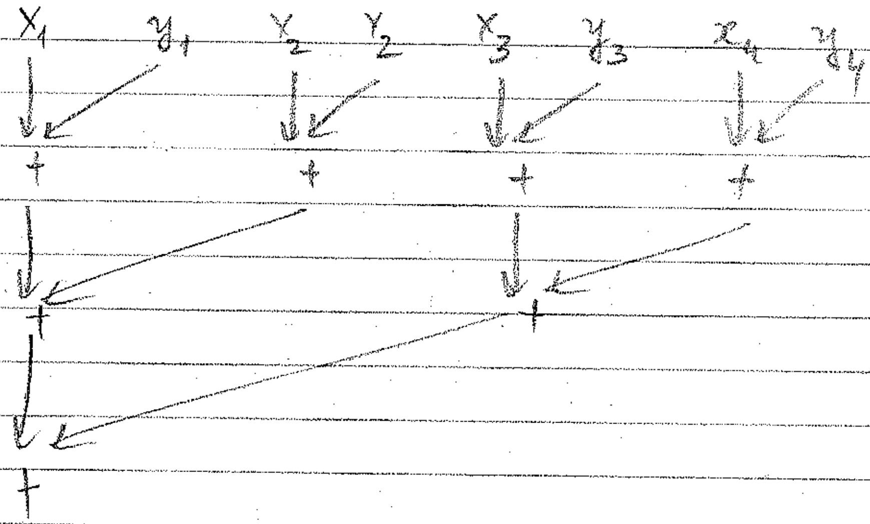
$n = 2^k \quad t = \log_2 n \quad p = \frac{n}{2}$

$k =$

Dot product :

$X_1, X_2 \cdots X_n$ $\qquad$ $X.Y = X_1 \times Y_1 + X_2 . Y_2 + \cdots + X_n Y_n .$

$Y_1, Y_2 \cdots Y_n$



$X_1 + \cdots + X_n \qquad p = \frac{n}{2}, \quad t = \log_2 n$

$X_1 . y_1 + x_2 . y_2 + \cdots + x_n . y_n \qquad t = 1 + \log_2 m, \; p = m$

$c_{ij} = a_{i1} . b_{1j} + a_{i2} . b_{2j} + \cdots + a_{im} . b_{mj}$

For each C_{ij}, we need $\log_2 m + 1$ time
computing all elements of C_{ij} is

$t = \log_2 m + 1 \qquad p = m^3$ processor.

$= m^2 \times m$

m^2 elements $\qquad$ C_{ij} : 1 element size of input $n = 2m^2$

$$\begin{pmatrix} a_{11} & \dots & a_{1m} \\ a_{21} & \dots & a_{2m} \\ a_{31} & \dots & a_{3m} \\ \dots & & \dots \end{pmatrix} \times \begin{pmatrix} b_{11} & \dots & b_{1m} \\ b_{21} & \dots & b_{2m} \\ b_{31} & \dots & b_{3m} \\ \dots & & \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

<u>Def:</u> A problem is parallelizable if we can solve it in polylog time on polynomial # of processors.

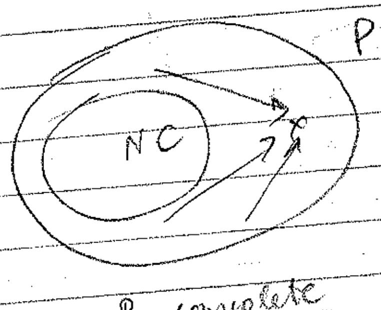
$t \leq P(\log n)$

$\boxed{NC}$ Nick's class

Nickel = <u>devil</u> iron
German.

If we have p - processors, t - cycles $\} \rightarrow$ time $= t \times p$ sequentialized

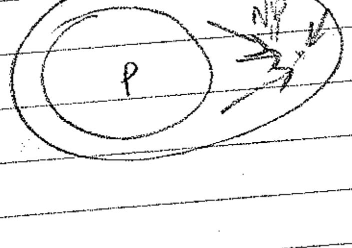
time $\leq P_1(n)$



$\boxed{NC \subseteq P}$

$P \stackrel{?}{=} NC$

Unknown.



$NP = P^?$

NP.complete.

Exam : P- complete.

Problem : Linear program

P- complete

Find solutions to systems of linear inequalities.

rice	meat	apples	bread.	milk
X_1	X_2	X_3	X_4	X_5

kg

calories →

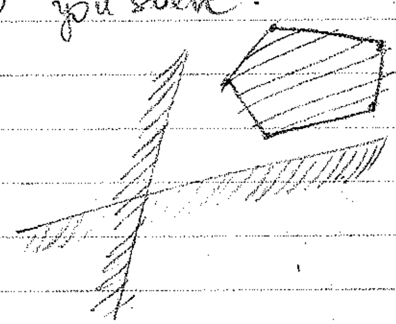
Calories : $2{,}000 X_1 + 4{,}000 x_2 + 100 x_3 + 1000 X_4 + 150 X_5 \geq 2{,}500$

Proteins $0 \cdot X_1 + 1 \cdot X_2 + 0 \cdot x_3 + 0.1 X_4 + 0.05 X_5 \geq 0.03$

$0.1 X_1 + 0 \cdot x_2 + 1 \cdot x_3 + 0.1 X_4 + 0.01 \cdot X_4 \geq 0.01$

Price : $1.7 x_1 + 5.0 x_2 + 2 x_3 + 2 \cdot X_4 + 0.75 \cdot X_5 \leq 4.0$

How do you solve ?



Linear Programming

Khachiyan 1978 LP $\subseteq$ P

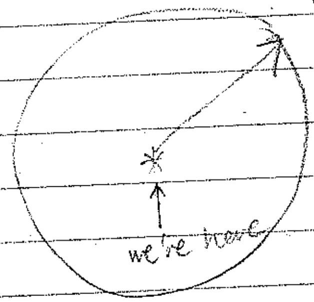
N. Karmarkar 1984 :
invented a practical poly time algorithm.

Theorem : Linear Programming is P-complete.

Ideal world, when we only take into account comp. time $t \sim \log n$

in practice ??

$R = c \cdot T_{par}$



$$V_{proc} \leq \frac{V}{\Delta V} = \frac{\frac{4}{3}\pi c^3}{\Delta V} = T_{par}^3$$

$$T_{seq} \leq V_{proc} \cdot T_{par} \leq c \cdot T_{par}^3 \cdot T_{par}$$

$$T_{par}^4 \geq c \cdot T_{seq}$$

We have limitations

1) $v \leq c$
2) $V = \frac{4}{3}\pi R^3$ Euclidean Geometry

↓

real space-time is curved.