$IC \in P^{SAT}$ — an oracle

$NP \equiv P^{NP}$

If SAT can be solved with ORACLE then IC can be solved in polytime w.r.t. SAT (oracle)

ORCLE gives ans in 1 step.

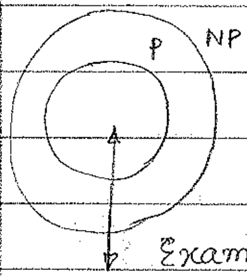
$\Pi_2 P = \Pi_1 P$ means we can solve $\Pi_2 P$, if we solved $\Pi_1 P$.

Qs. whether, $NP^{A} \stackrel{?}{=} P^{A}$ where A is ORACLE

Nobody can prove NP=P by diagonization, which is a ORACLE.

Tuesday 21st April

Inside P



Problem :- Polynomial time may still mean too long.

Solution :- distribute, parallize.

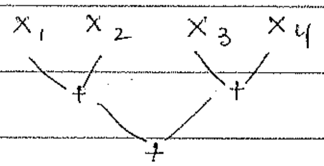
Example of parallization :-

$x_1 + x_2 + x_3 + \dots + x_n$

let's we have 2 processors,

Pr 1: $x_1 + \dots + x_{n/2}$ Pr 2: $x_{n/2+1} + \dots + x_n$ ~~needs 2 cycles~~

finally ~~add them together~~

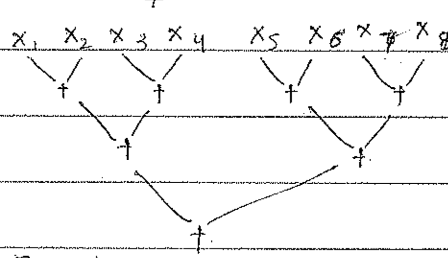


~~2 cycles~~

$n=4$: $t=2$ $p=2$

$n=8$ $t=3$ $p=4$ *

$n=2^k$ $t=\log_2 n$ $p=n/2$



3 cycles

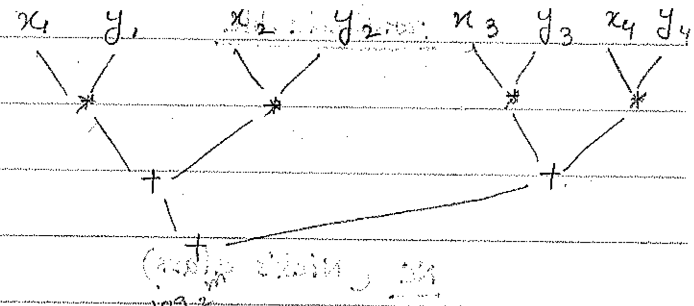
For product, this is exactly same as *.

Qs. 16 variables product — determine *

dot product

$x_1, \ldots, x_m$

$y_1, \ldots, y_m$



$x \cdot y = x_1 \cdot y_1 + \cdots + x_m \cdot y_m$

$t = 1 + \log_2 m$

$p = m$

For the whole product,

$n = 2m$, $t = \log n$, $p = n/2$

$\log 2m = \log 2 + \log m = 1 + \log m$

$$\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ a_{21} & \cdots & a_{2m} \\ \vdots & & \\ a_{m1} & \cdots & a_{mm} \end{pmatrix} \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ b_{21} & \cdots & b_{2m} \\ \vdots & & \\ b_{m1} & \cdots & b_{mm} \end{pmatrix} = \begin{pmatrix} c_{11} & \cdots & c_{1m} \\ c_{21} & \cdots & c_{2m} \\ \vdots & & \\ c_{m1} & \cdots & c_{mm} \end{pmatrix}$$

$c_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + \cdots + a_{im} * b_{mj}$

for each c_{ij}, we need $(1 + \log_2 m)$ time

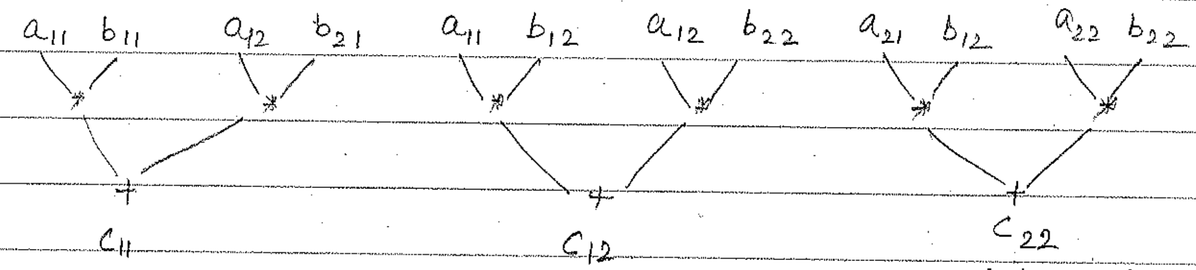
m processors.

Since we do ||ly,

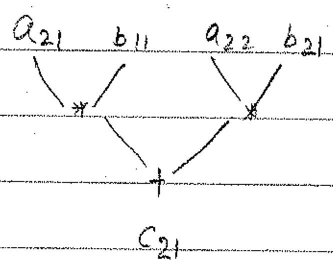
computing all elements of c_{ij} is :-

$t = 1 + \log_2 m$, $p = m \cdot (m^2) = m^3$ processors [since m^2 elements]

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$



n [input size]



$n = 2m^2$

$m \sim n^{1/2}$

$t \sim \log n$ $\quad p \sim n^{3/2}$

Parallelizable

A problem is parallelizable if we can solve it in polylog time on polynomial # of processors.

$t \leq P(\log n)$

NC (Nick's class)

If we have p processors t cycles (for sequential calculation)

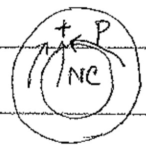
time : $t * p$. $p \leq P_1(n)$

$t \leq P_2(\log n)$

$\therefore$ multiplication is also bounded by polynomial.

$\boxed{NC \subseteq P}$ so, NC is subset of P.

$P \stackrel{?}{=} NC$



P-complete = problems hard in P & if they can be parallelize, then everything else can be ".

Ex. of P-complete problem: linear programming (LP)

Finding solutions to systems of linear inequalities

	rice	meat	apples	bread	milk
foods amt	x_1	x_2	x_3	x_4	x_5
Calories/kg	2000	4000	100	1000	150

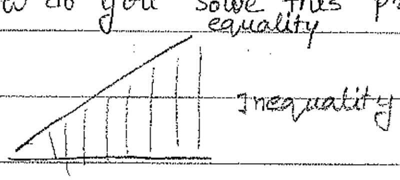
calories: $2000x_1 + 4000x_2 + 100x_3 + 1000x_4 + 150x_5 \geq 2500$

protein: $0.1x_1 + 1\cdot x_2 + 0\cdot x_3 + 1\cdot x_4 + .05x_5 \geq 0.3$

vitamin: $0.1\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 + 0.1\cdot x_4 + .01\cdot x_5 \geq 0.1$

Cost: $1.7x_1 + 5x_2 + 2\cdot x_3 + 2\cdot x_4 + .75\cdot x_5 \leq \$4.0$ (To meet this conclusion, we can find $x_1, \dots x_5$)

How do you solve this problem? We try 2^n possible variations.



Simplex method : 2^n time is required to solve LP.

Khachiyan solved $LP \subseteq P$

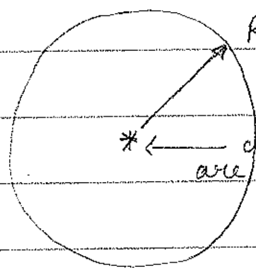
N. Karmakar (1984) – invented a practical polynomial time algorithm.

Theorem :– LP is P-complete.

Ideal world, when we take into a/c comp-time $t \sim \log n$, parallel.

In practise ??

is $T_{seq} \Rightarrow T_{par}$



$R = c \cdot T_{par}$ $\quad V = \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi c^3 \cdot T_{par}^3$

$N_{processor} \leq \frac{V}{\Delta v} = \frac{\frac{4}{3}\pi c^3}{\Delta v} T_{par}^3$ → constant = c

$T_{seq} \leq N_{processor} \cdot T_{par} \leq c \cdot T_{par}^3 \cdot T_{par}$

$\leq c \cdot T_{par}^4$.

$T_{par}^4 \geq c \cdot T_{seq}$.

$T_{par} \geq c \cdot T_{seq}^{1/4}$

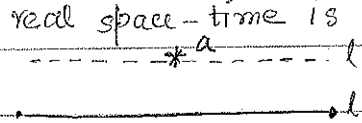
$T_{par} \geq c \cdot n^{1/4}$ (It's not good as $\log n$)

we have limitations, caused by

1. $v \leq c$.
2. $V = \frac{4}{3}\pi R^3$ Euclidean geometry.

real space-time is curved.

V th postulate.



There is one line ‖ to l through point 'a'