Fuzzy Logic and Computation with words

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Inspiration

- Real world is messy

- Computing with words is inspired by the remarkable human capability to perform a wide variety of physical and mental tasks without any measurements and any computations.

Computation "Stack"

- Crisp sets
- Fuzzy sets
- Computing with words (CWW)

Crisp sets

- Distinct values
  - x is 1 or 0
- Set
  - S = {x | x meets some conditions}
- Membership function
  - μ_S = { 1 if x $\in$ S

    { 0 if x $\notin$ not element of S

Crisp sets don't always work

- Sorites Paradox
  - Take 1,000,000 grains of sand to be a heap
  - Remove a grain

- What's temperature is cold?

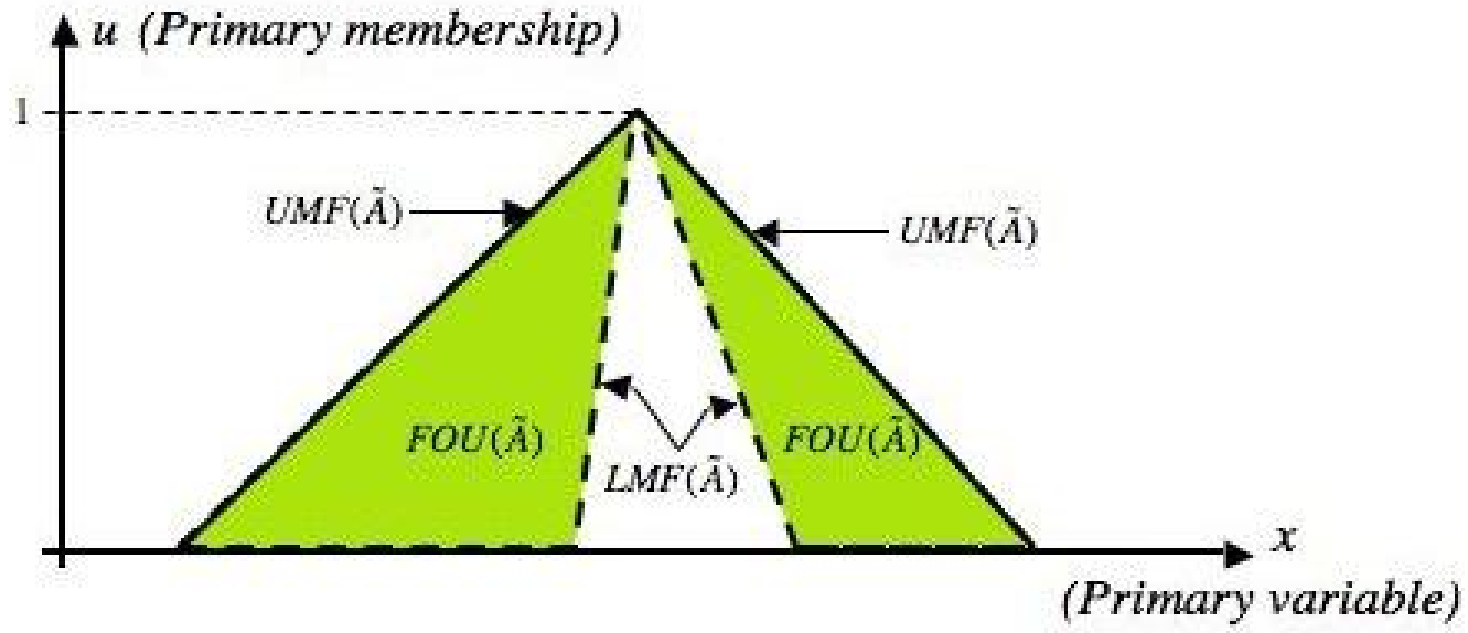
Fuzzy Sets

- Variables with soft values
  - Value of x is imprecise

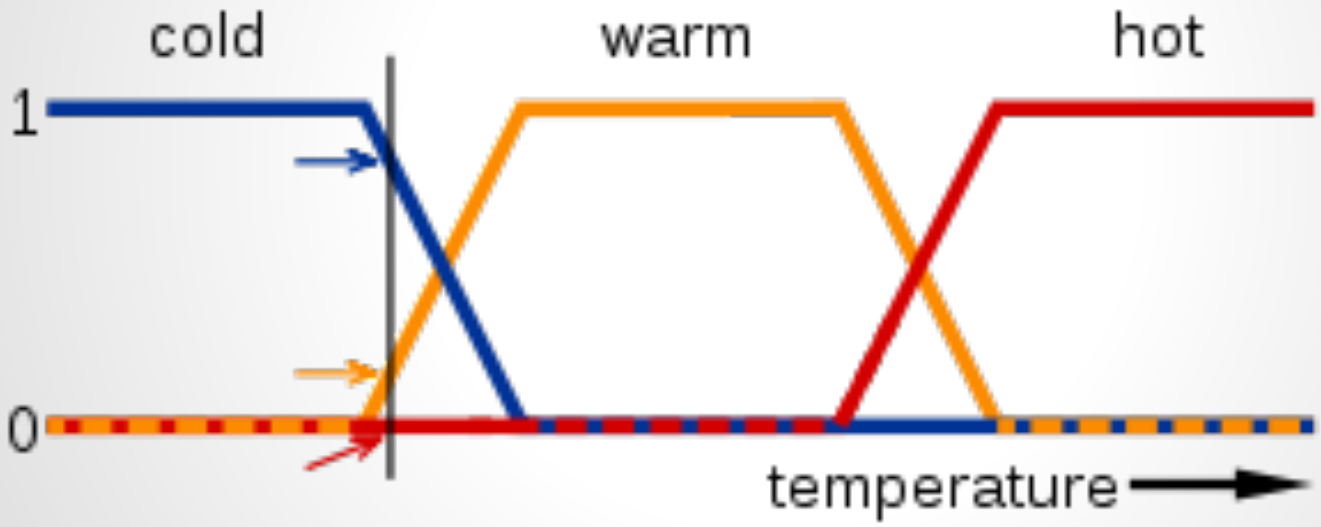
- Set
  - Contains pairs (U:m)
  - Set S = {(x_0:0.4), (x_1:0.2), (x_2:0.2), ...}

- Membership function
  - $S = \{(x, m(x)) \mid x \in X\}$

Fuzzy Set


u (Primary membership)
1
UMF(Ã)
UMF(Ã)
FOU(Ã)
FOU(Ã)
LMF(Ã)
x
(Primary variable)

Example of fuzzy sets

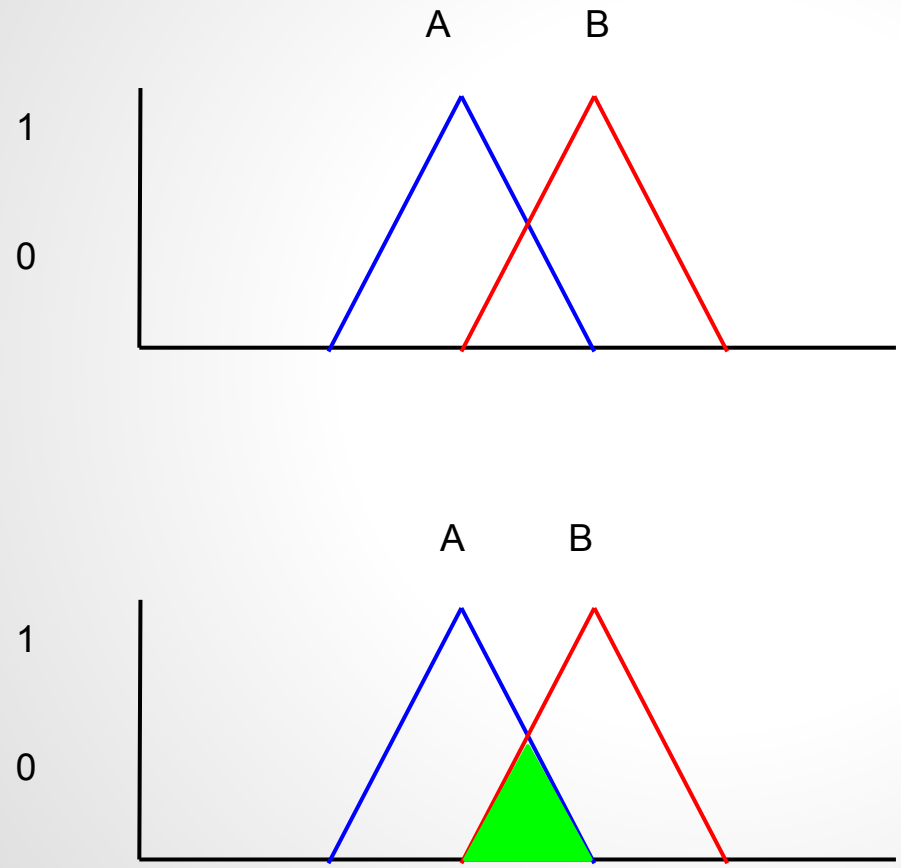


Fuzzy Set Operators

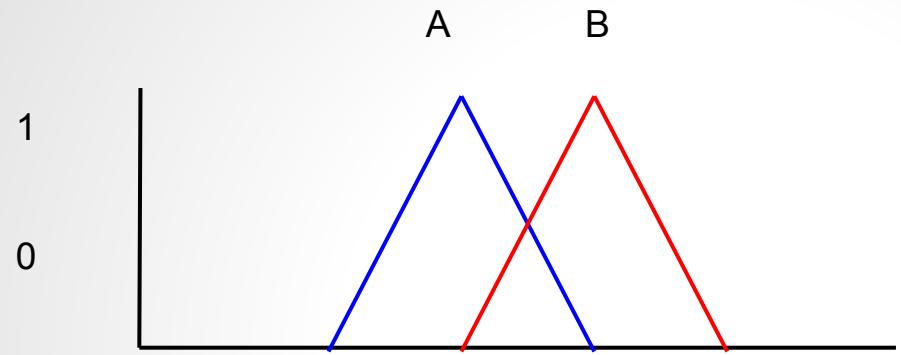
- Same operators as regular sets
  - And, Or, Not

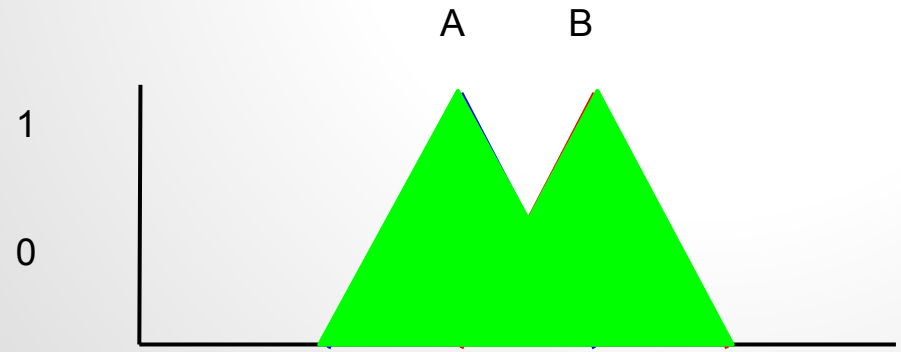
- Can be defined in different ways

And

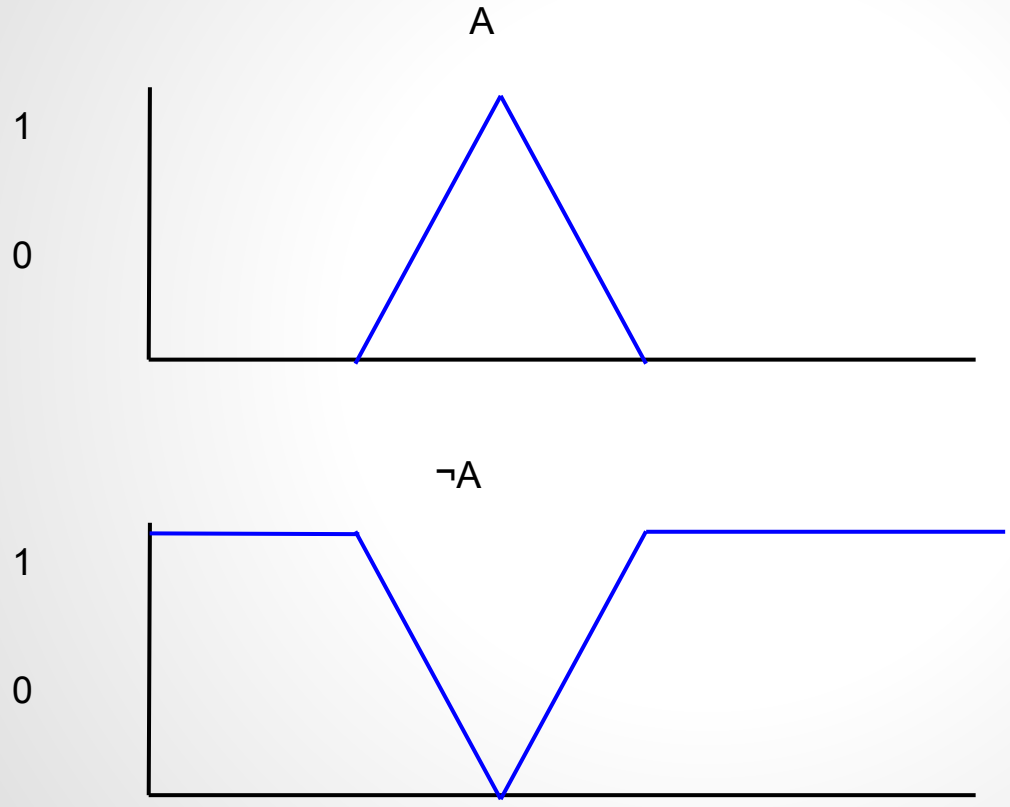


Or





Not

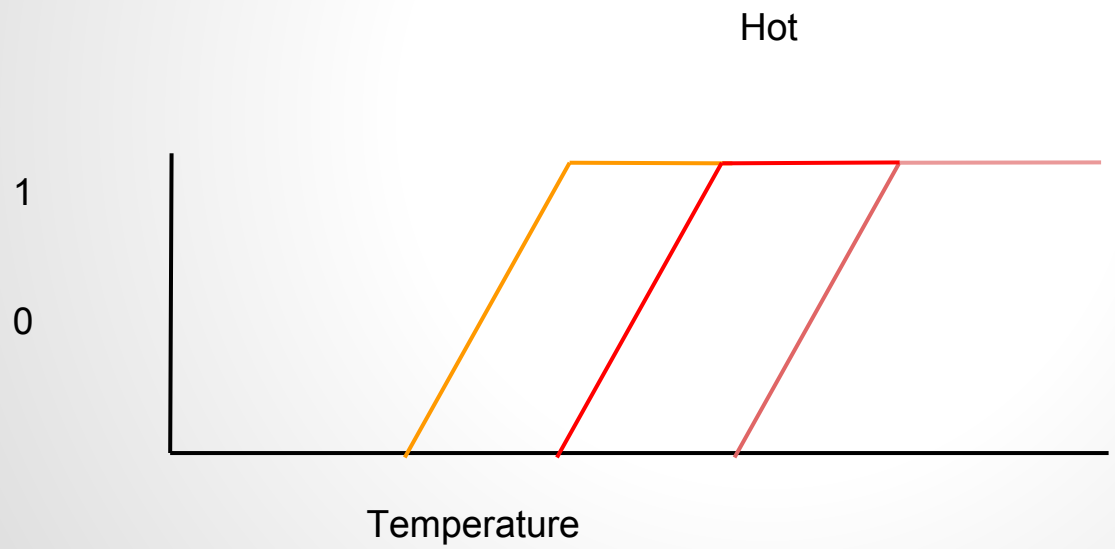


Fuzzy Logic

- Uses linguistic variables
- Series of if then rules
- More than one rule can apply
- Can hedge variables

Hedge

- Modifier: very, a little



Apply rules

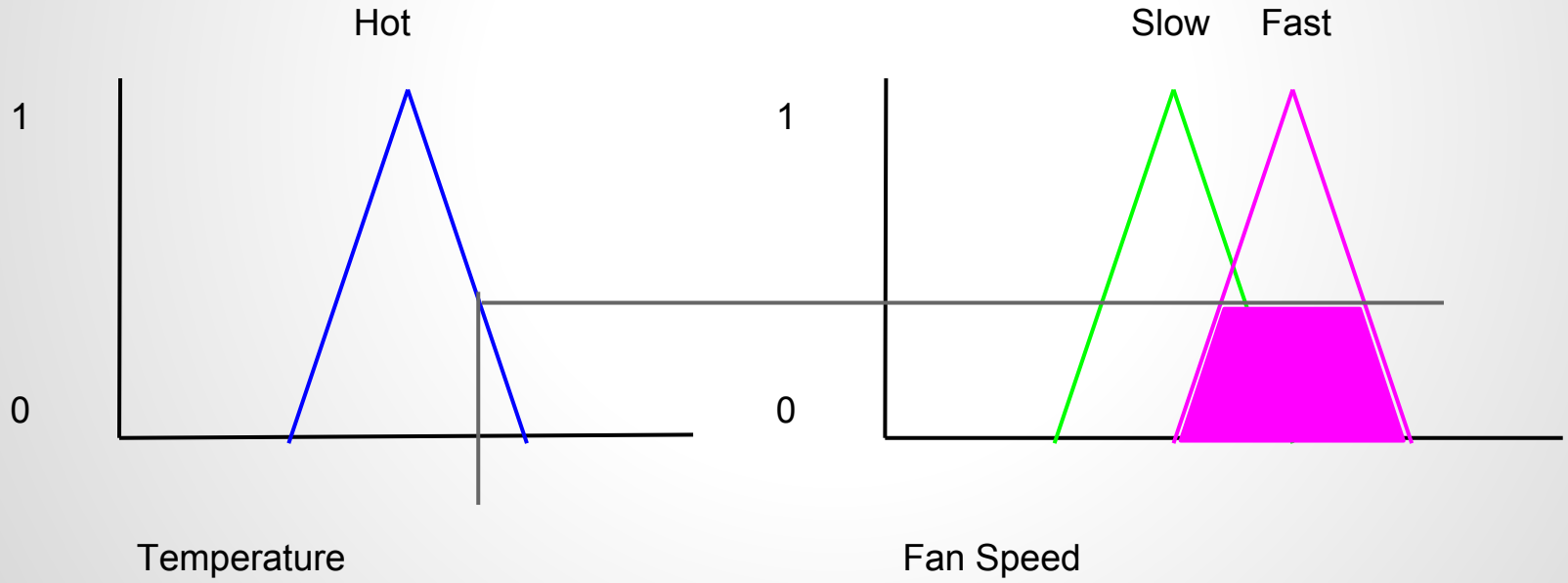
- All rules evaluated
- Results applied simultaneously

Fuzzification

- Crisp inputs need to be fuzzified
- Fuzzy outputs need to be defuzzified

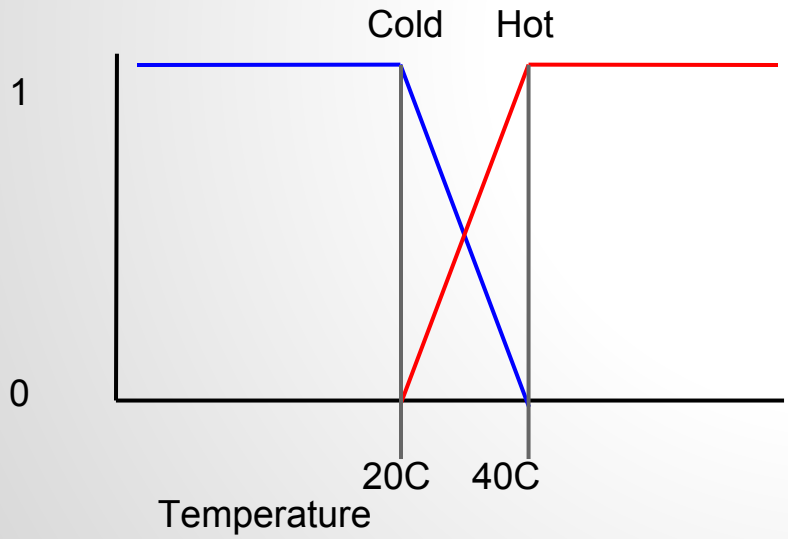
Example

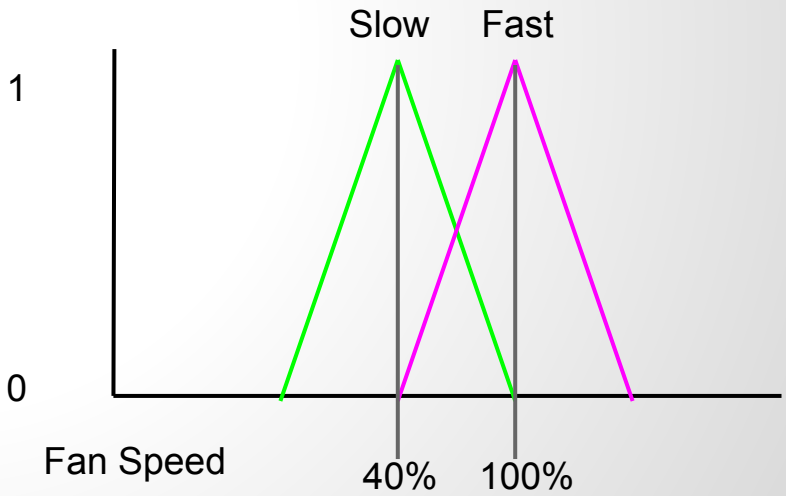
If (temperature is "hot") Then (fan is fast)



Example

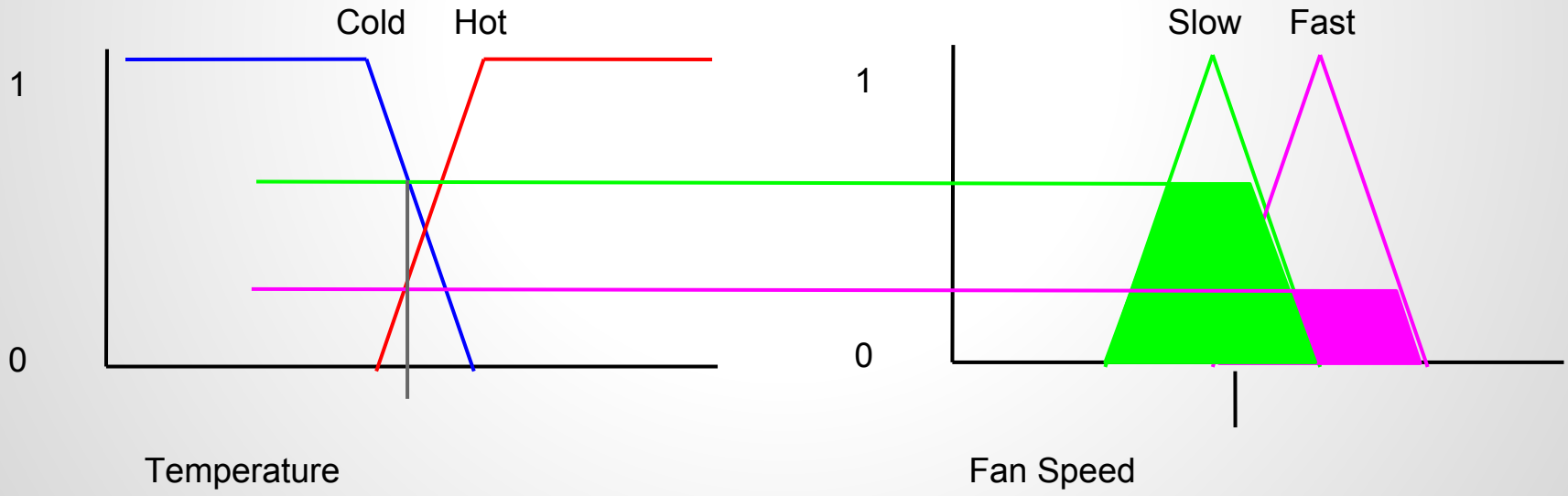
If (temperature is "hot") Then (fan is fast)
If (temperature is "cold") Then (fan is slow)





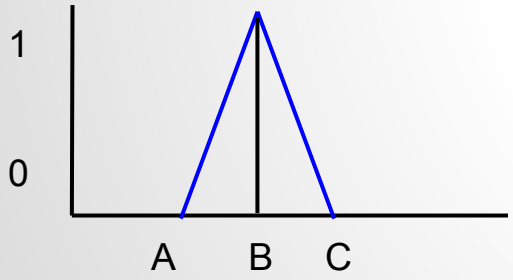
Example

Temperature = 26C



Triangle Membership Function

- Easy to use
- Only one dimension



$$triangle(x, a, b, c) = \begin{cases} 0, x \leq a \\ \frac{x-a}{b-a}, a \leq x \leq b \\ \frac{c-x}{c-b}, b \leq x \leq c \\ 0, c \leq x \end{cases}$$

Membership grades

- Cold:
  - For $20 < t < 40$ $m(t) = (40-t)/(40-20)$
  - $t = 24$ $m(t) = .7$

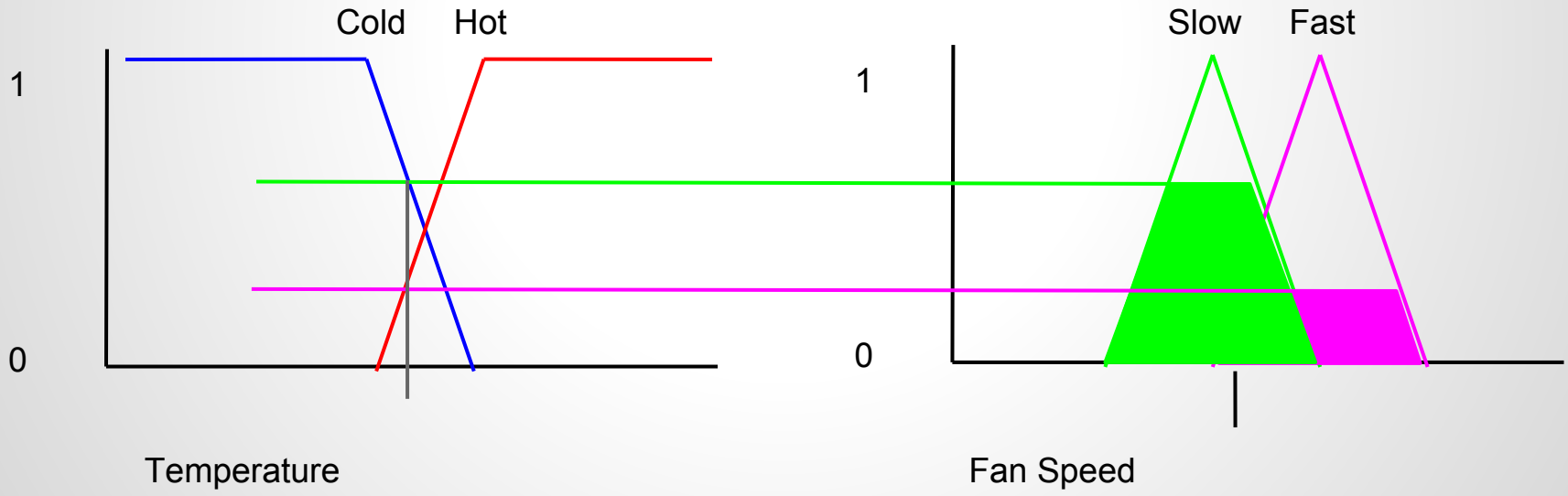
- Hot:
  - For $20 < t < 40$ $m(t) = (t - 20)/(40-20)$
  - $t = 24$ $m(t) = .3$

Inputs

- As a result of fuzzification our inputs are now cold at .7 and hot at .3

Example

Temperature = 26C



Output defuzzification

- Outputs are taken from inputs
  - Slow is .7
  - Fast is .3
- If we only want one or the other (best fit)
  - Fan speed = max(slow,fast) = slow = 20%
- We can take the fan speed over the interval from 40%-50%
- Fuzzy weighted average
  - Fan speed = $(m_{slow} * .4 + m_{fast} * 1)/ (m_{slow} + m_{fast})$
  - Fan speed = .7 * .4 + .3 * 1 = .58 = 58%

Output Defuzzification

- Centroid or Center of Gravity are commonly used
- Takes the center of the combined area
- Can be estimated by taking an average of points

Hedge

- Same calculation but with rule that has hedge
- If (temperature is " very cold") Then (fan is slow)
- For this case very = t - 5 (the hedge calculation can vary)

Membership grades

- Cold = .7 Hot = 3
- s
- To apply hedge we add 5to t (of course we can also subtract 5 from the intervals)

- For $20 < t + 5 < 40$ $m(t) = (40-t + 5)/(40-20)$

- For $t = 26$ $m(t) = .95$

Output with hedge

- Using fuzzy weighted average
- $(m_{off}*0+m_{slow}*.4+m_{fast}*1)/(m_{off}+m_{slow}+m_{fast})$
- (.95 * 0 + .7 * .4 + .3 * .1)/(.95 + .7 + .3)
- Fan speed = .30 = 30%

Computing with words

- Granules
- Propositions
- Example: Mary is young

Process

- Take Initial Data Set
- Use Fuzzy logic to make inferences
- Terminal Data Set is result

Explanatory Database

- Maps propositions to Fuzzy variables
- Mary is young
- ED = Population[Name; Age] + Young[Age;μ]
- $x = {}_{Age}\text{POPULATION}\ [\text{Name} = \text{Mary}]$

Constraint Propagation

- Allows new constraints to be inferred
  - X is A
  - X is B
  - X is A ∪ B

- Extends existing information

Advantages

- Simplifies control systems
- Works with imprecise data
- Mesh with reality better

Further work

- Building rules
- Limited IDS/TDS, ED
- Adding variables increases rules exponentially