Computing With Words

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Problem

- To describe a quantity using the words such as small, medium, high, etc. We get a crude description of the quantity

- More accurate description: Short but closer to medium. To a large extent, short but to some degree medium height.

Problem

- Fix words W_1, … W_n each quantity is then represented by tuple of degrees $d=(d_1, \ldots d_n)$
- If we have quantity *q* represented by *d* and quantity *q'* represented by *d'*. We would like to produce:

$$q+q', q-q', q*q' \textit{ etc.}$$

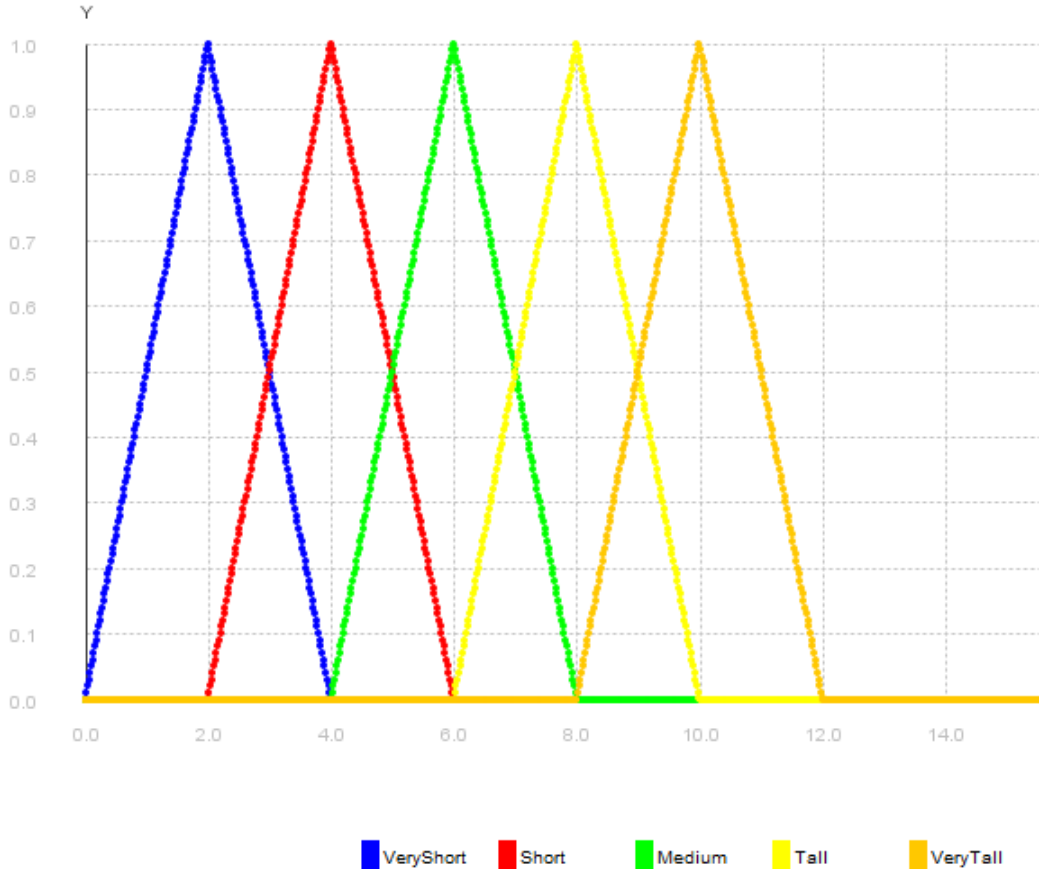
- Instead of computing with numbers, we should be able to compute with words.

Idea

- Fuzzy logic
- To each word W_i assign a function $M_i(x)$, that describes for each value of x, the degree to which this values satisfies the corresponding property
- Functions $M_i(x)$ differ by a shift.

Idea

- $M_i(x)$ are usually triangular, first linearly decreasing from 0 to 1. The linearly decreasing from 1 to 0.

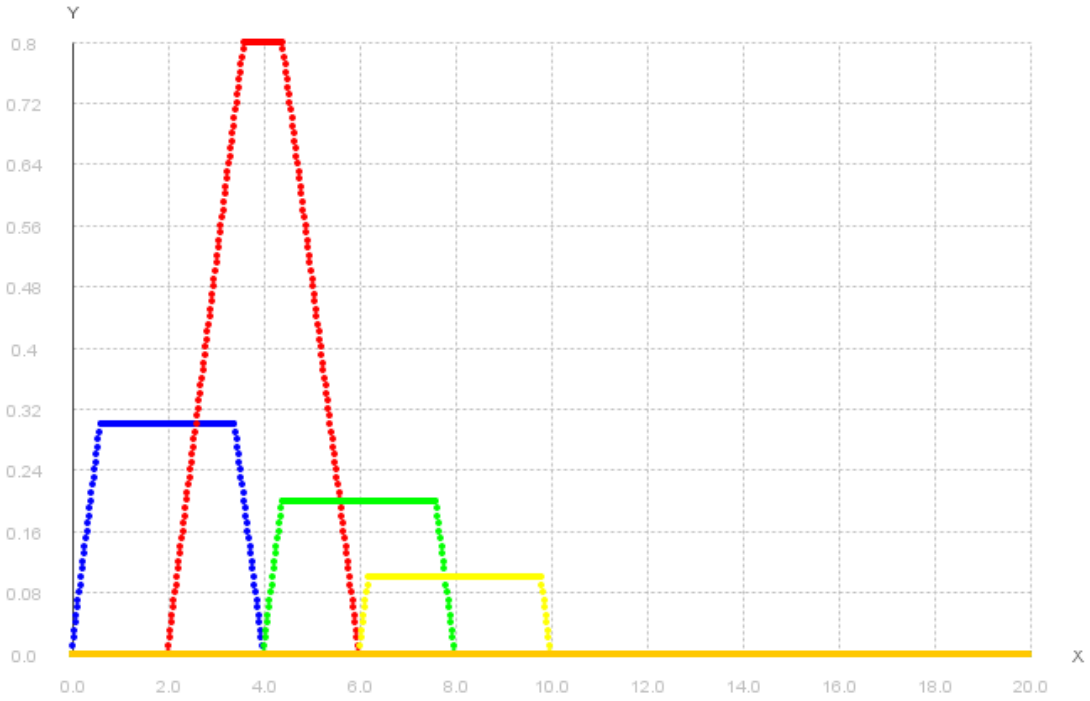


$$Mi(x) = \max(0,1 - \frac{|x - (s + i * h)|}{h})$$

Idea

- The degree to which q is characterized by the word W_i and x satisfies the property described by this word can be described as

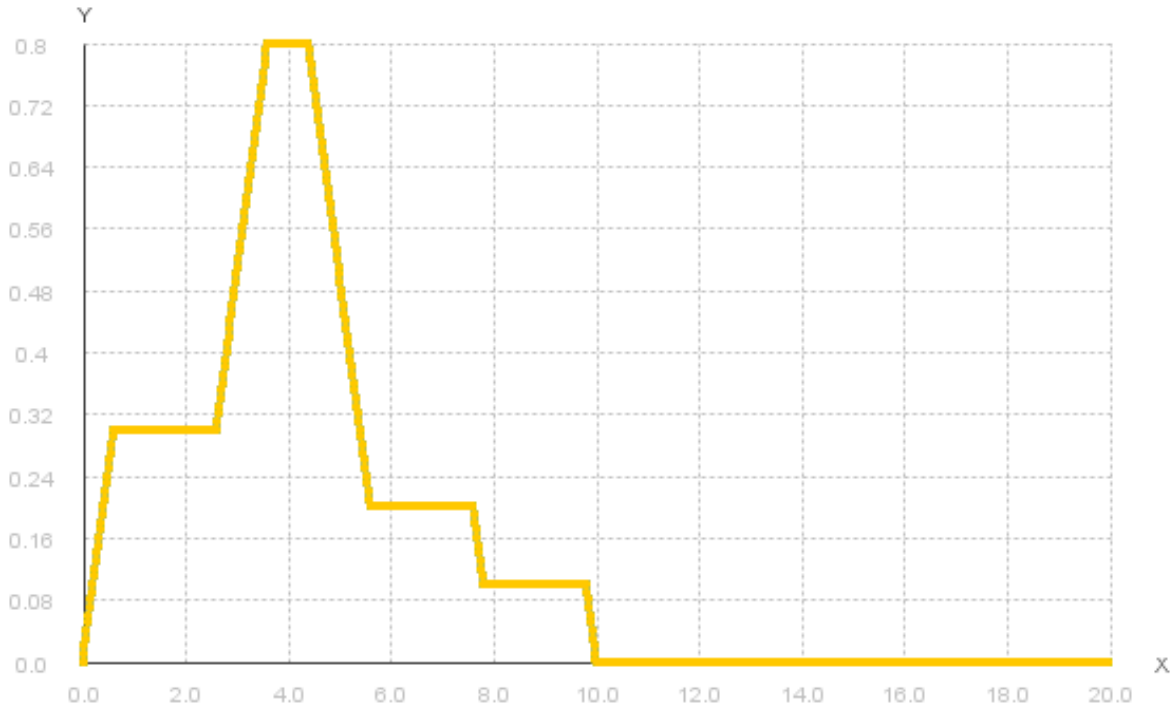
$$\min(d_i, M_i(x))$$



Idea

- The degree to which one of these conditions is satisfied is equal to

$$M_d(x) = \max(\min(d_1, M_1(x)), \ldots, \min(d_n, M_n(x)))$$

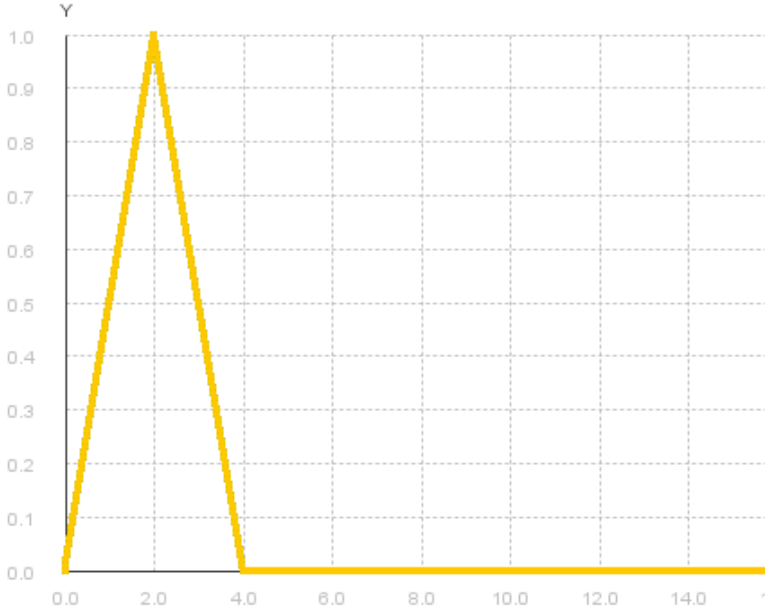


How to compute f(a, b)

For tuples *d* and *d'* describing 2 quantities *q* and *q'*

- Generate membership functions $M_d(x)$ and $M_{d'}(x)$

Medium



Very Short

How to compute f(a, b)

- By applying Zadeh's extension principle to these membership functions, we can compute:

Addition

$$M_{x1+x2}(y) = \max x\,(\min(M_1(x), M_2(y-x)))$$

Subtraction

$$M_{x1-x2}(y) = \max x\,(\min(M_1(x), M_2(x-y)))$$

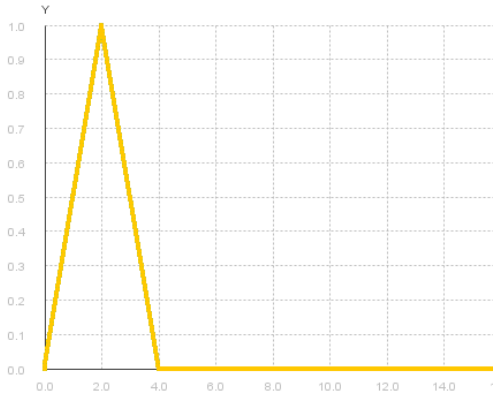
Multiplication

$$M_{x1*x2}(y) = \max x\,(\min(M_1(x), M_2(y/x)))$$

How to compute f(a, b)

Addition

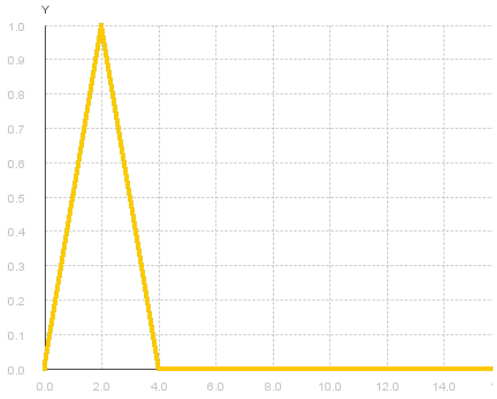
$$M_{x1+x2}(y) = \max x\,(\min(M_1(x), M_2(y-x)))$$

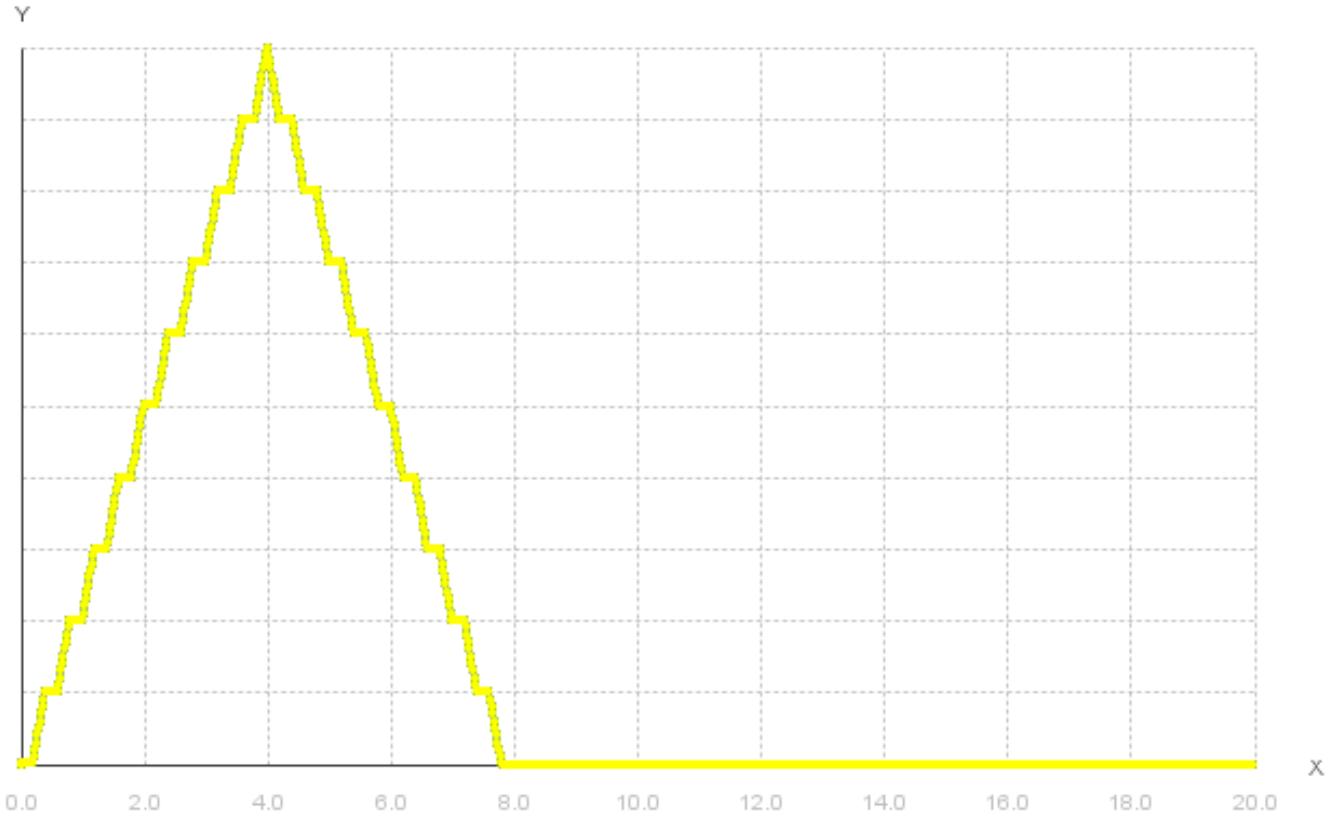


How to compute f(a, b)

Subtraction

$$M_{x1\text{-}x2}(y) = \max x\,(\min(M_1(x), M_2(x\text{-}y)))$$

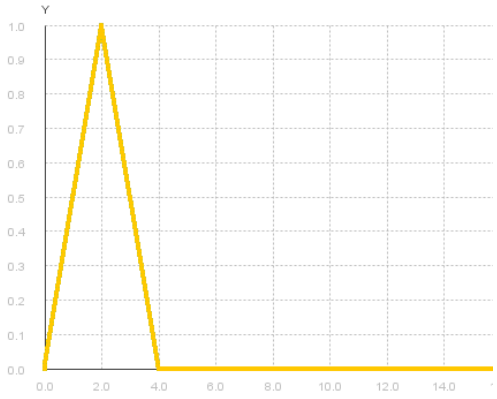


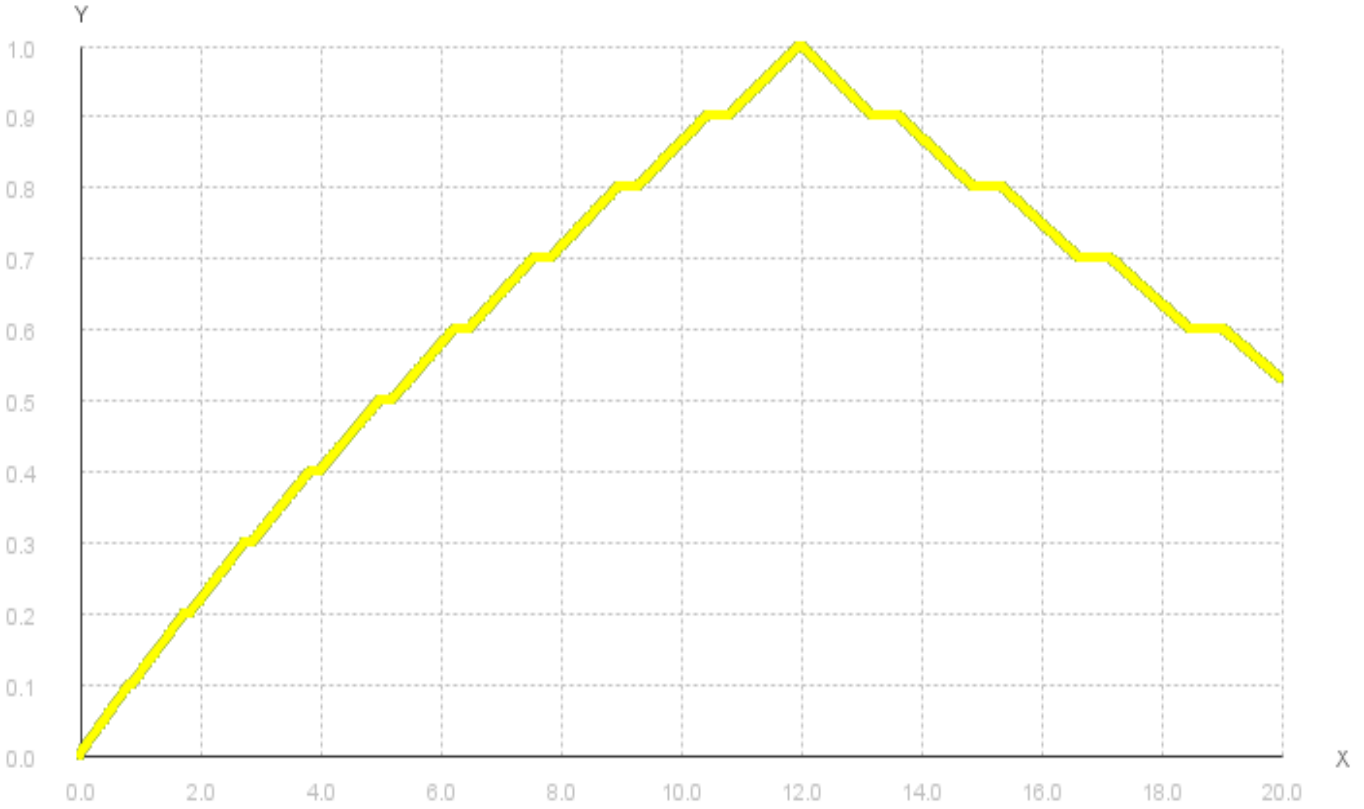


How to compute f(a, b)

Multiplication

$$M_{x1*x2}(y) = \max x\,(\min(M_1(x), M_2(y/x)))$$

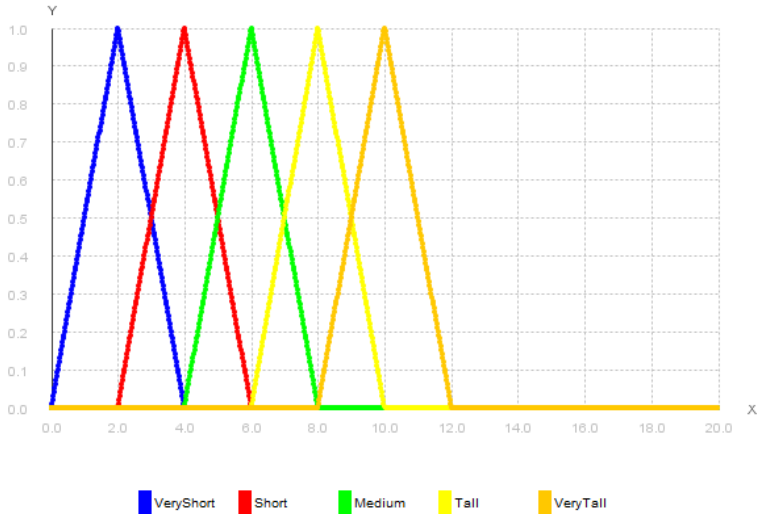


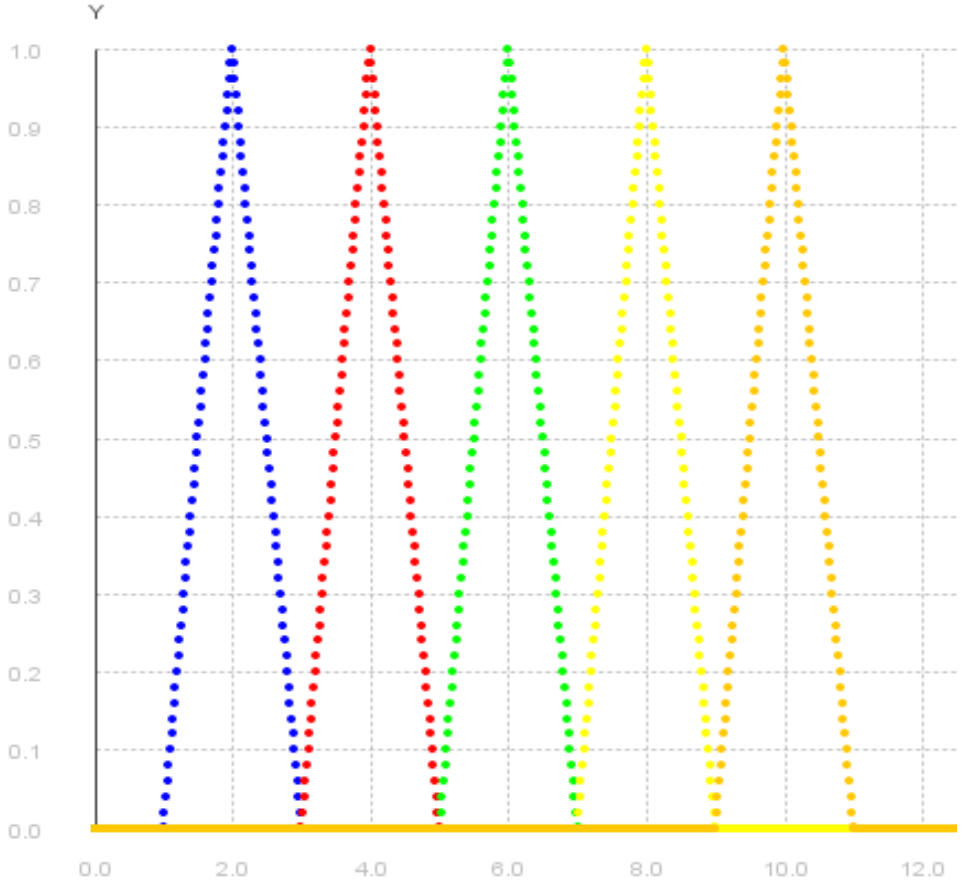


How to retrieve a tuple from a membership function

$$A_i(x) = \max(1-|x-i|, 0)$$

$$A'_i(x) = \max(A_i(x) - A_{i-1}(x) - A_{i+1}(x), 0)$$





How to retrieve a tuple from a membership function

$$M'(x) = \max(M(x) - A_{i-1}(x) - A_{i+1}(x), 0)$$

$$d_i = \max x\,(\min(A'_i(x), M'(x)))$$

```
d0 of Membership Function of M1 + M2 0.0
d1 of Membership Function of M1 + M2 0.0
d2 of Membership Function of M1 + M2 0.4999999999999991
d3 of Membership Function of M1 + M2 0.9999999999999987
d4 of Membership Function of M1 + M2 0.4999999999999982
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M1 = Medium

M2 = Very Short

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d0 of Membership Function of M1 - M2 0.4999999999999991
d1 of Membership Function of M1 - M2 0.9999999999999987
d2 of Membership Function of M1 - M2 0.4999999999999982
d3 of Membership Function of M1 - M2 0.0
d4 of Membership Function of M1 - M2 0.0
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d0 of Membership Function of M1 * M2 0.21739130434782605
d1 of Membership Function of M1 * M2 0.4000000000000008
d2 of Membership Function of M1 * M2 0.5769230769230768
d3 of Membership Function of M1 * M2 0.714285714285714
d4 of Membership Function of M1 * M2 0.862068965517241
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