[redacted]



Lec-03

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**p.r:**

+ , * , − , / , % , if-then, or, and,
= , ≠ , <, ≤ , ≥ , >,

$$a \dot{-} b = \max(a - b, 0) = \begin{cases} a-b & \text{if } a \geq b \\ 0 & \text{otherwise} \end{cases}$$

Instead of −−,

$$\text{prev}(n) = \begin{cases} n-1 & \text{if } n \geq 1 \\ 0 & \text{otherwise} \end{cases} = n \dot{-} 1$$

so, $\begin{cases} \text{prev}(0) = 0 \\ \text{prev}(m+1) = m \end{cases}$

$$\begin{cases} f(0) = 0 \\ f(m+1) = m \end{cases}$$

$$\begin{cases} f(0) = g(\ ) \\ f(m+1) = h(m, f(m)) \end{cases}$$

$g = 0, \quad h = \pi^2_1$

so, $\text{prev} = PR(0, \pi^2_1)$

**~~Th~~ $Th^m$:** prev is p.r.

**$Th^m$-2:** $\dot{-}$ is p.r

**Proof:** $a \dot{-} b = a \underbrace{\dot{-} 1 \dot{-} 1 \ldots \dot{-} 1}_{b \text{ times}}$

```
int sub = a ;
for (int i=1 ; i<=b ; i++)
{ sub = prev (sub) ; sub -1 ;
```

$\boxed{14}$

~~sub(std~~

$sub(a, 0) = a$

$sub(a, m+1) = prev(sub(a, m))$

$f(n_1, 0) = n_1$

$f(n_1, m+1) = prev(f(n_1, m))$

$f(n_1, 0) = g(n_1)$

$f(n_1, m+1) = h(n_1, m, f(n_1, m))$

$g(n_1) = n_1 = \pi^1_1$

$h = prev(f(n_1, m))$

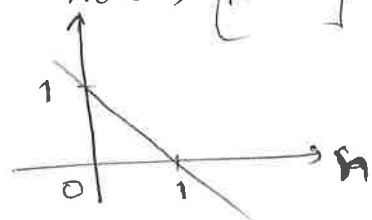
$= prev(\pi^3_3)$

$\dot{-} = PR(\pi^1_1, prev(\pi^3_3))$

Java boolians are: not , and , or
! && ||

not (0) = 1
not (1) = 0

not(n) $[y = 1 - x]$




so, $not(n) = 1 \dot{-} n$
$= \sim(0) \dot{-} n$

not is p.r.

and (a, b) → In engineering, it is multiplication

| b \ a | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

and is p.r.

or (a, b) → it's addition

| b \ a | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | |

~~$a \vee b = 7(7a \,\&\, 7b)$~~

$a \vee b = 7(7a \,\&\, 7b)$ → de Morgan law

or (a, b) = not (and (not(a), not(b)))

$= 1 \dot{-} (1 \dot{-} a) * (1 \dot{-} b) = 1 - (1-a)(1-b)$

$= 1 - (1 - a - b + a*b)$

$= a + b \dot{-} a*b$

not is p.r
and " p.r
so, or is p.r

16

$$\text{eqto } 0(n) = \begin{cases} \text{true} & \text{if } n = 0 \\ \text{false} & \text{otherwise} \end{cases}$$

$\text{eq to } 0(0) = 1$ $\qquad f(0) = 1$

$\text{eq to } 0(m+1) = 0$ $\qquad f(m+1) = 0$

$f(0) = g(\ )$

$f(m+1) = h(m, f(m))$

$g = 1, \quad h = 0$

$= \sigma \circ 0$

$\text{eq to } 0 = PR(\sigma \circ 0, 0)$

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If $a = 0$, how will we define $a = b$?

$a \dot{-} b = 0 \Leftrightarrow a \leq b,$

$a = b \Rightarrow a \dot{-} b$

bcs $\quad a \dot{-} b = \begin{cases} a - b, & \text{if } a > b \\ 0, & \text{otherwise} \end{cases}$

$a \leq b \Leftrightarrow \text{eq to } 0(a \dot{-} b)$

so, $\leq$ is p. r.

$a = b \Leftrightarrow \underbrace{a \leq b}_{\text{p.r.}} \ \&\& \ \underbrace{b \leq a}_{\text{p.r.}}$

so, $=$ is p. r.

$a \neq b \Leftrightarrow\ !(a = b)$

so, $\neq$ is p. r.

$a < b \Leftrightarrow a \leq b \ \&\& \ !\, a = b$

so, $<$ is p. r.

 $n\%4$

$0 \% a$

$$\begin{cases} rem(a, 0) = 0 \\ rem(a, m+1) = \begin{cases} rem(a, m)+1 & \text{if } rem(a,m)+1 < a \\ 0 & \text{otherwise} \end{cases} \end{cases}$$

| $m$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... |
|---|---|---|---|---|---|---|---|---|---|---|
| $m\%4$ | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 | 0 | ... |

**Th$^m$:** If $P, g, h$ are p.r. then
if $P(\bar{n})$ then $g(\bar{n})$ else $h(\bar{n})$ is also p.r.

Brainstorm:
- either $P$ is true and we have $g(\bar{n})$
or $P$ is false & " " $h(\bar{n})$

$abs(n) = (n >= 0) : n \; ? \; -n$

$P(\bar{n}) * g(\bar{n}) \; + \; (1 - P(\bar{n})) * h(\bar{n})$

If $P(\bar{n})$ is true, then $g(\bar{n})$
" $P(\bar{n})$ " false, " $h(\bar{n})$

So, if - then - elser is p.r.

if $(rem(a,m)+1 < a)$ $\{rem(a,m)+1\}$
else $(0)$

So, rem is p.r.

18

$m/a$

$$\mathrm{div}(a,0) = 0$$

$$\mathrm{div}(a,m+1) = \begin{cases} \mathrm{div}(a,m)+1 & \text{if } \mathrm{rem}(a,m+1)=0 \\ \mathrm{div}(a,m) & \text{otherwise} \end{cases}$$

| $m$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $m\%4$ | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 | 0 | 1 | ... |
| $m/4$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | ... |

HW-2

1. if $p$ then $f$, else if $q$ then $g$ else $h$

2. Prove from scratch that reminder is p.r.