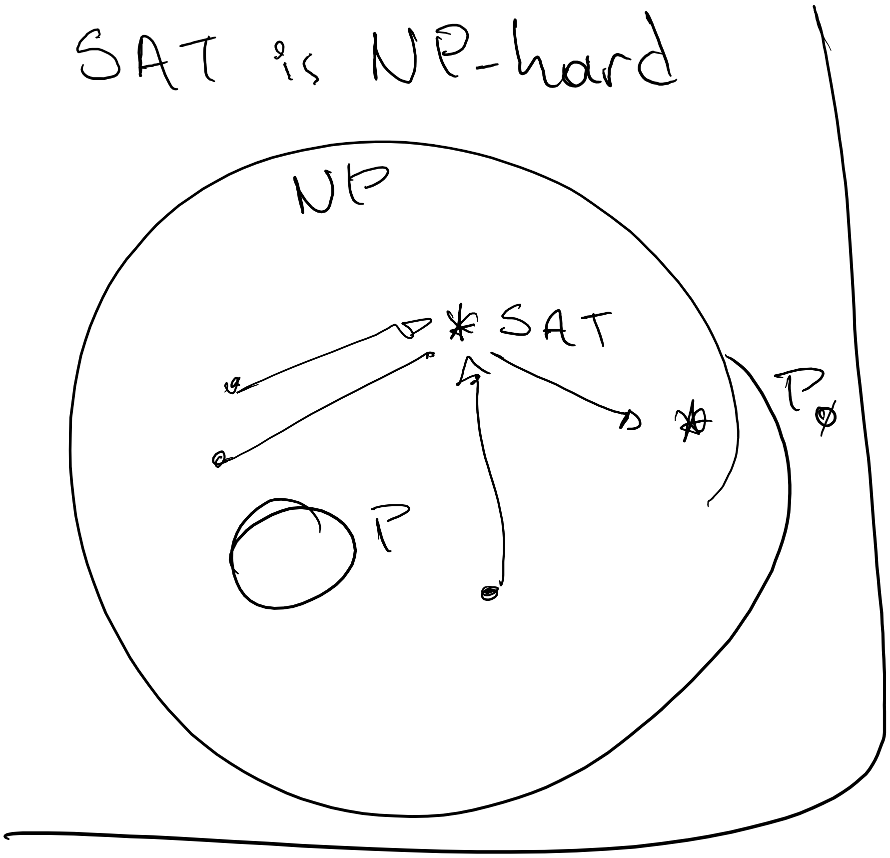
Apr 11

Tuesday, April 11, 2017 3:02 PM

SAT is NP-hard



3-SAT
CNF-form
with ≤ 3
variables per
clause

$(x_1 \vee x_2) \wedge (x_1 \vee \neg x_2 \vee x_3)$
&
....

Th. 3-SAT is NP-hard

Proof: we reduce SAT to 3-SAT

$$\begin{tikzcd} X \arrow[r] \arrow[d] & y \text{ - SAT vector} \\ X' \arrow[r] & y' \arrow[u] \end{tikzcd}$$

3-SAT

Describing v_1:

$a \vee (\neg b \,\&\, c)$

Parse expression:

$r_1 = \neg b$

$r_2 = r_1 \& c$

$r_3 = a \cup r_2$

$(r_1 = \neg b) \& (r_2 = r_1 \& c) \& (a \vee r_2)$

r_1	b	F	$\neg F$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$\neg F \equiv (\neg r_2 \& \neg b) \vee (r_1 \& b)$

$F \equiv (r_1 \vee b) \& (\neg r_1 \vee \neg b)$

$r_2 = r_1 \& c$

r_1	c	r_2	F	$\neg F$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

$\neg F = (\neg r_1 \,\&\, \neg c \,\&\, r_2) \vee$
$(\neg r_1 \,\&\, c \,\&\, r_2) \vee$
$(r_1 \,\&\, \neg c \,\&\, r_2) \vee$
$(r_1 \,\&\, c \,\&\, \neg r_2)$

$F \equiv (r_1 \vee c \vee \neg r_2) \,\&$
$(r_1 \vee c \vee r_2) \,\&$
$(\neg r_1 \vee c \vee \neg r_2) \,\&$
$(\neg r_1 \vee \neg c \vee r_2)$

$(r_1 \vee b) \,\&\, (\neg r_1 \vee \neg b) \,\&\, (r_1 \vee c \vee \neg r_2) \,\&$
$(r_1 \vee c \vee r_2) \,\&\, (\neg r_1 \vee c \vee \neg r_2) \,\&\, (\neg r_1 \vee \neg c \vee r_2) \,\&$
$(a \vee r_2)$

CNF Conjunction of disjunctions

$C_1 \,\&\, C_2 \,\&\, \ldots$

where

$C_i = a \vee b \vee \ldots$ called clause

$(a \vee b) \,\&\, (\neg a \vee \neg b)$ = in CNF

$\neg(a \vee b) \vee \neg a$ = not in CNF

3-color problem

Given: a graph

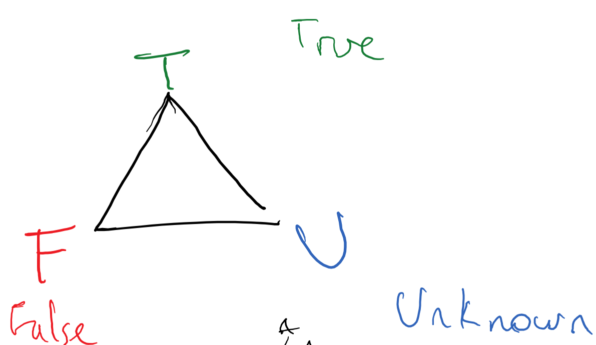
Find: coloring in 3 colors so that neighbors have different colors

Proof: Reduce 3-SAT to this problem

$(x_1 \vee x_2) \,\&\, \underline{(\overline{x_1} \vee \overline{x_2} \vee x_3)}$

palette

r

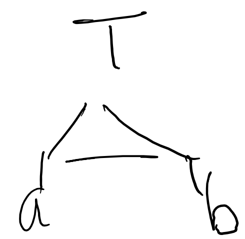


$x_1 \to \neg x_1$

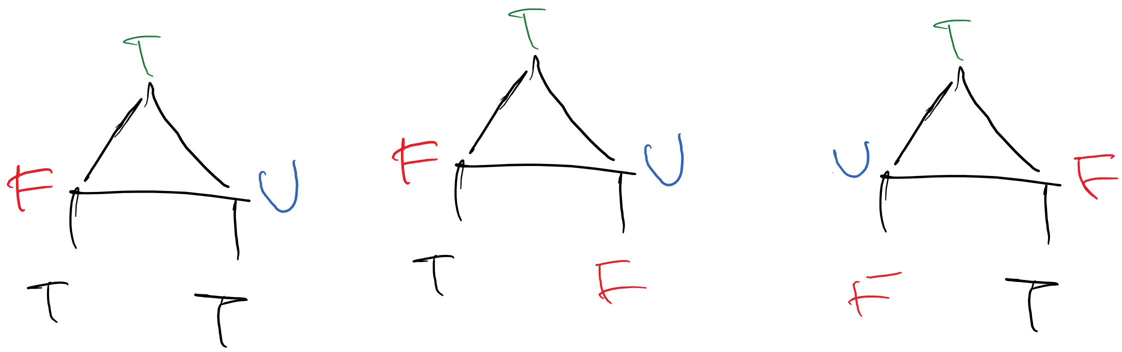
$x_2 \to \neg x_2$

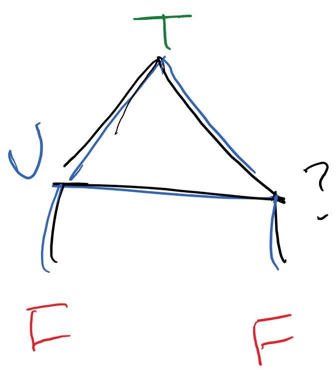
$x_3 \to \neg x_3$

"or"-gadget

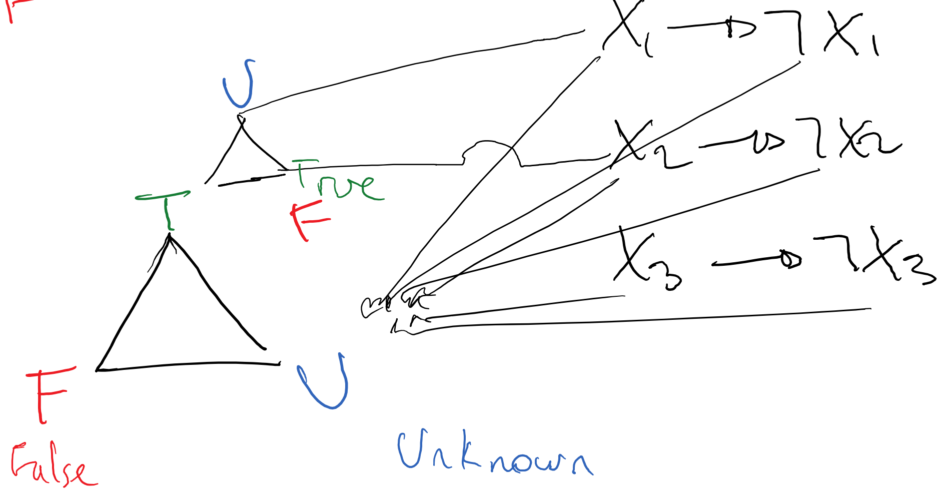


claim: gadget can be colored $\Longleftrightarrow$ $a \vee b$ is true



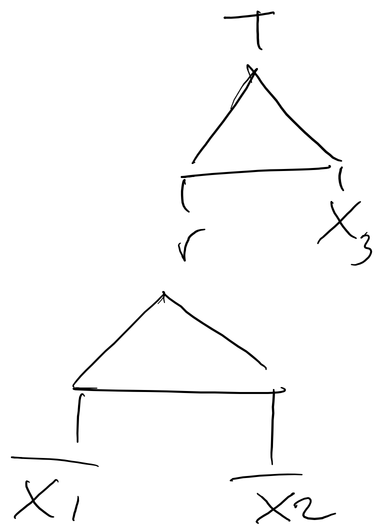


← We can't color this.



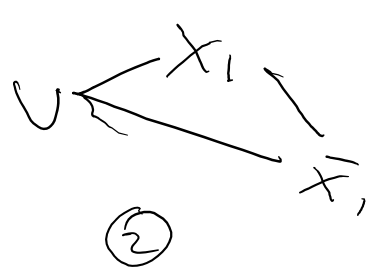
$r = \neg x_1 \vee \neg x_2$

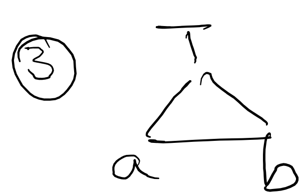
$r \vee x_3$



Algorithm

1. Start w/ palette
2. add variables & their negations
3. for clauses $a \vee b$, add an or-gadget
4. for clauses $a \vee b \vee c$, add





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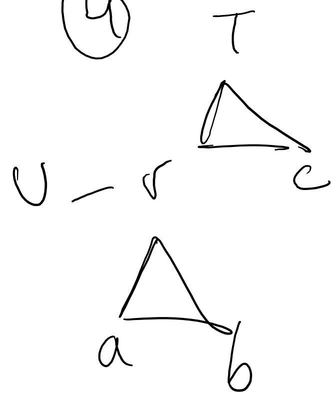
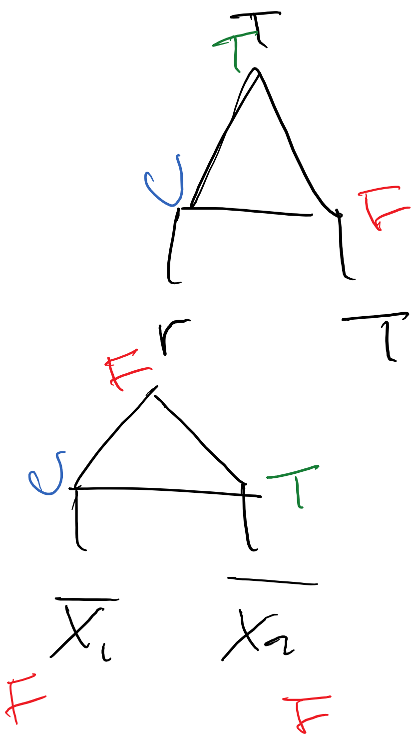
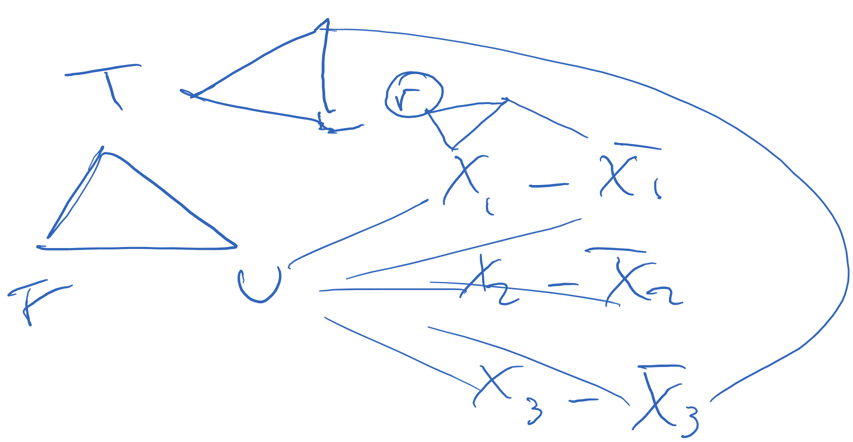

T
U
r
c
a
b


T
T
U
F
r
T
F
U
T
$\overline{X_1}$
$\overline{X_2}$
F
F


T
r
F
U
X_1 — $\overline{X_1}$
X_2 — $\overline{X_2}$
X_3 — $\overline{X_3}$

Clique ⇒ Node that is connected to all other nodes.

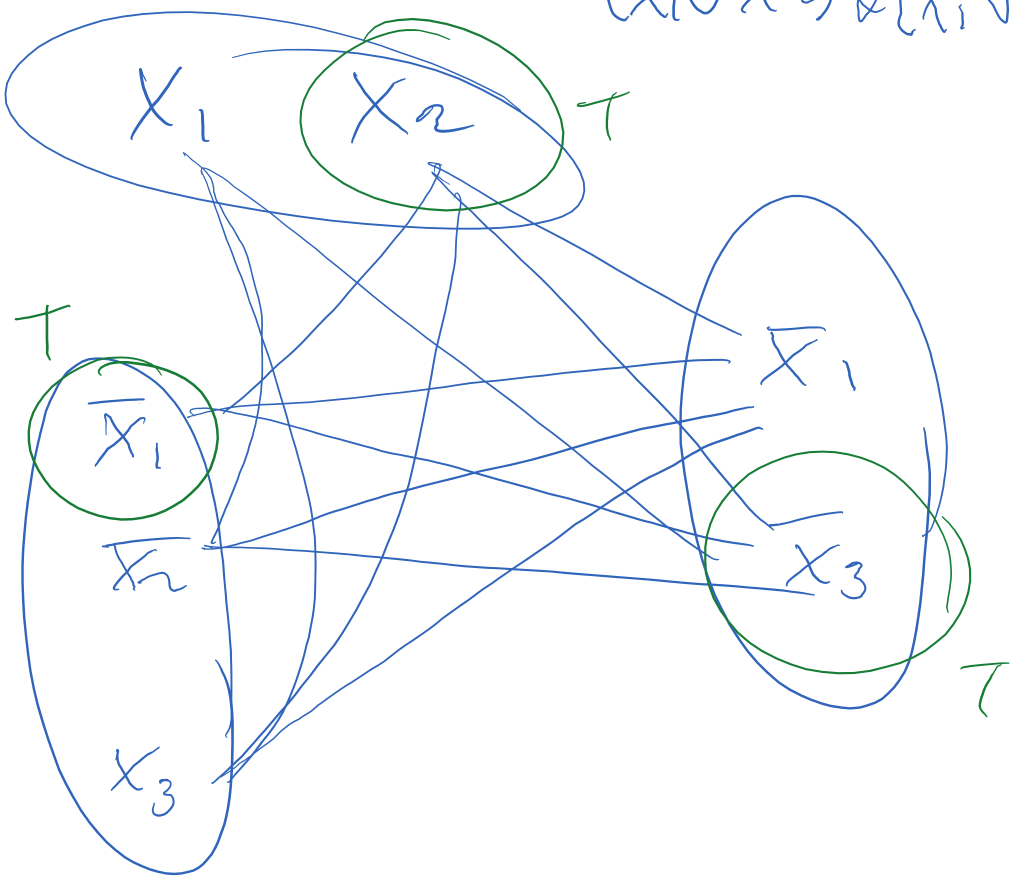
Problem:

Given a graph G and an integer K

Find:

A clique of size K

$(x_1 \vee x_2) \,\&\, (\bar{x}_1 \vee \bar{x}_2 \vee x_3) \,\&\, (\bar{x}_1 \vee \bar{x}_3)$



*Connect nodes from dif. clauses, with only exceptions: never connect variable to its negation

$X_1 = T$

$X_2 = T$

$X_3 = F$