Solution to Homework 31

Homework 31. On the example of the formula $(a \vee b \vee \neg c \vee \neg d) \,\&\, (\neg a \vee \neg b \vee \neg c)$, show how checking its satisfiability can be reduced to checking satisfiability of a 3-CNF formula.

Solution. Following the general algorithm, we introduce a new variable r_1 meaning $a \vee b$, and replace the original formula with the formula

$$(a \vee b = r_1) \,\&\, (r_1 \vee \neg c \vee \neg d) \,\&\, (\neg a \vee \neg b \vee \neg c).$$

Then, we replace the part $a \vee b = r_1$ with the corresponding CNF formula

$$(a \vee b \vee \neg r_1) \,\&\, (a \vee \neg b \vee r_1) \,\&\, (\neg a \vee b \vee r_1) \,\&\, (\neg a \vee \neg b \vee r_1).$$

The resulting formula is:

$$(a \vee b \vee \neg r_1) \,\&\, (a \vee \neg b \vee r_1) \,\&\, (\neg a \vee b \vee r_1) \,\&\, (\neg a \vee \neg b \vee r_1) \,\&$$

$$(r_1 \vee \neg c \vee \neg d) \,\&\, (\neg a \vee \neg b \vee \neg c).$$