

Solution to Homework 37

Homework 37. If we take into account communication time, how fast can you compute the product of n numbers in parallel?

Solution. According to Section 2 of the corresponding handout, if we can solve the problem in parallel in time T_{parallel} , then we can also solve it sequentially in time $T_{\text{sequential}} \leq c \cdot T_{\text{parallel}}^4$. For computing the product of n numbers sequentially, the smallest possible time is $n - 1$: we need $n - 1$ multiplications. Thus,

$$n - 1 \leq c \cdot T_{\text{parallel}}^4.$$

Dividing both sides by c , we get $c^{-1} \cdot (n - 1) \leq T_{\text{parallel}}^4$, hence

$$T_{\text{parallel}} \geq C \cdot (n - 1)^{1/4} = C \cdot \sqrt[4]{n - 1}.$$

This is faster than the sequential time $n - 1$, but much slower than the time $\text{const} \cdot \log(n)$ that we would have if we ignored communication time.