Solution to Problem 22

Problem. Give:

- an example of computation time $t_A(x)$ for which the algorithm is practically not feasible, but is feasible according to the existing definition, and
- an example of computation time $t_A(x)$ for which the algorithm is practically feasible, but is not feasible according to the existing definition.

These examples must be different from the ones we had in class.

Solution.

First example: $t_A(x) = 10^{2021}$.

- This is a constant – so it is feasible in the sense of the formal definition.
- However, this number is larger than the number of particles in the Universe, and thus, not practically feasible.

Second example: $t_A(x) = \exp(10^{-2021} \cdot \text{len}(x))$.

- This function is exponentially growing – thus, not feasible in the sense of the formal definition, since every exponential function grows faster than a polynomial.
- However, for every input x of realistic length $\text{len}(x)$, we have

  $$10^{-2021} \cdot \text{len}(x) \ll 1,$$

  thus, $t_A(x) = \exp(10^{-2021} \cdot \text{len}(x)) < \exp(1) = e = 2.71828\ldots$, i.e., $t_A(x) = 1$ or $t_A(x) = 2$, a very feasible number of computational steps.