

Solution to Homework 26

Problem. On the example of the formula $(\neg a \vee \neg b \vee c \vee \neg d) \& (\neg a \vee \neg b \vee \neg c)$, show how checking its satisfiability can be reduced to checking satisfiability of a 3-CNF formula.

Solution. Following the general algorithm, we introduce a new variable r_1 meaning $\neg a \vee \neg b$, and replace the original formula with the formula

$$(\neg a \vee \neg b = r_1) \& (r_1 \vee c \vee \neg d) \& (\neg a \vee \neg b \vee \neg c).$$

Then, we replace the part $\neg a \vee \neg b = r_1$ with the corresponding CNF formula

$$(\neg a \vee \neg b \vee \neg r_1) \& (\neg a \vee b \vee r_1) \& (a \vee \neg b \vee r_1) \& (a \vee b \vee r_1).$$

The resulting formula is:

$$\begin{aligned} &(\neg a \vee \neg b \vee \neg r_1) \& (\neg a \vee b \vee r_1) \& (a \vee \neg b \vee r_1) \& (a \vee b \vee r_1) \& \\ &(r_1 \vee c \vee \neg d) \& (\neg a \vee \neg b \vee \neg c). \end{aligned}$$