Solution to Homework 27

Problem. On the example of the formula $(\neg a \vee b \vee \neg c) \,\&\, (a \vee \neg b)$, show how checking its satisfiability can be reduced to coloring a graph in 3 colors.

Solution. According to the general algorithm, first, we build a palette: three vertices T, and F, and U all connected to each other. Then, we:

- add two vertices a and $\neg a$, and connect both of them to U and to each other;
- add two vertices b and $\neg b$, and connect both of them to U and to each other;
- add two vertices c and $\neg c$, and connect both of them to U and to each other.

For the 3-literal clause $C_1 = \neg a \vee b \vee \neg c$, we:

- add a new vertex $\neg a \vee b$,
- we connect this vertex to U,
- we add an or-gadget for $(\neg a \vee b) \vee \neg c$, i.e., we add new vertices $(\neg a \vee b)_1$ and $\neg c_1$, connect both these two vertices to T and to each other, and connect $(\neg a \vee b)_1$ to $a \vee b$ and $\neg c_1$ to $\neg c$;
- we add vertices $\neg a_1$ and b_1 and connect them to the vertex $\neg a \vee b$ and to each other, and
- we connect $\neg a_1$ to $\neg a$ and b_1 to b.

After that, for the 2-literal clause $C_2 = a \vee \neg b$, we add an or-gadget: namely,

- we add two new vertices a_2 and $\neg b_2$,
- we connect both vertices a_2 and $\neg b_2$ to T and to each other, and
- we connect a to a_2 and $\neg b$ to $\neg b_2$.