Solution to Problem 31

Problem. Show how to compute the sum of 11 numbers in parallel if we have unlimited number of processors. How many processors do we need and how much time will the computation take? Why do we need parallel processing in the first place?

Solution. Suppose that we are given 11 numbers $x_1, \ldots, x_{11}$, and we want to compute their sum. Then:

- at the first moment of time:
  - the first computer computes $x_1 + x_2$,
  - the second computer computes $x_3 + x_4$,
  - the third computer computes $x_5 + x_6$,
  - the forth computer computes $x_7 + x_8$,
  - the fifth computer computes $x_9 + x_{10}$;
- at the second moment of time:
  - the first computer computes the sum $x_1 + x_2 + x_3 + x_4$ as
    $$(x_1 + x_2) + (x_3 + x_4);$$
  - the second computer computes the sum $x_5 + x_6 + x_7 + x_8$ as
    $$(x_5 + x_6) + (x_7 + x_8);$$
  - the third computer computes the sum $x_9 + x_{10} + x_{11}$ as
    $$(x_9 + x_{10}) + x_{11};$$
- at the third moment of time, the first computer computes the sum
  $$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$
  as
  $$(x_1 + x_2 + x_3 + x_4) + (x_5 + x_6 + x_7 + x_8);$$

- at the fourth moment of time, the first computer computes the desired sum

  $$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

  as

  $$(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8) + (x_9 + x_{10} + x_{11}).$$

These computations require 5 computers and 4 moments of time.

In general, parallel computations are needed to speed up computations. Without parallelism, we would need 10 moments of time to compute the desired sum.