

Solution to Problem 7

Problem. Prove that the following function is mu-recursive:

```
int j = 1;
while(!(a * j <= m))
  {j++;}
```

Solution. According to the general algorithm, first, we write a similar problem, but with a for-loop:

```
int j = 1;
for(i = 1; i <= b; i++)
  {j++;}
```

This program can be translated into primitive recursion as follows:

$$j(0) = 1;$$

$$j(n+1) = j(n) + 1.$$

A general primitive recursion defines a function $h(n_1, \dots, n_k, m)$ of $k+1$ variables. In our case, we have a function of 1 variable, so $k+1 = 1$ and $k = 0$. For $k = 0$, the general primitive recursion has the form

$$j(0) = f;$$

$$j(n+1) = h(n, j(n)).$$

Here, $f(0) = 0$ and $h = \sigma \circ \pi_2^2$, so

$$j(m) = PR(0, \sigma \circ \pi_2^2).$$

As the number of iterations, we take the smallest n for which $a \cdot j(n) \leq m$, i.e., $\mu_n.(a \cdot j(n) \leq m)$. Thus, the desired function has the form

$$j(\mu_n.(a \cdot j(n) \leq m)).$$