Solution to Problem 31

Problem. Show how to compute the product of 11 numbers in parallel if we have unlimited number of processors. How many processors do we need and how much time will the computation take? Why do we need parallel processing in the first place?

Solution. Suppose that we are given 11 numbers $x_1, \ldots, x_{11}$, and we want to compute their product. Then:

- at the first moment of time:
  - the first computer computes $x_1 \cdot x_2$,
  - the second computer computes $x_3 \cdot x_4$,
  - the third computer computes $x_5 \cdot x_6$,
  - the forth computer computes $x_7 \cdot x_8$,
  - the fifth computer computes $x_9 \cdot x_{10}$;
- at the second moment of time:
  - the first computer computes the product $x_1 \cdot x_2 \cdot x_3 \cdot x_4$ as
    $$(x_1 \cdot x_2) \cdot (x_3 \cdot x_4);$$
  - the second computer computes the product $x_5 \cdot x_6 \cdot x_7 \cdot x_8$ as
    $$(x_5 \cdot x_6) \cdot (x_7 \cdot x_8);$$
  - the third computer computes the product $x_9 \cdot x_{10} \cdot x_{11}$ as
    $$(x_9 \cdot x_{10}) \cdot x_{11};$$
- at the third moment of time, the first computer computes the product
  $$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_6 \cdot x_7 \cdot x_8$$
  as
  $$(x_1 \cdot x_2 \cdot x_3 \cdot x_4) \cdot (x_5 \cdot x_6 \cdot x_7 \cdot x_8);$$

- at the fourth moment of time, the first computer computes the desired product

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_6 \cdot x_7 \cdot x_8 \cdot x_9 \cdot x_{10} \cdot x_{11}$$

as

$$(x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_6 \cdot x_7 \cdot x_8) \cdot (x_9 \cdot x_{10} \cdot x_{11}).$$

These computations require 5 computers and 4 moments of time.

In general, parallel computations are needed to speed up computations. Without parallelism, we would need 10 moments of time to compute the desired product.