Solution to Homework 32

**Problem.** If we take into account communication time, how fast can you compute \( n! \) numbers in parallel?

**Solution.** According to Section 2 of the corresponding lecture, if we can solve the problem in parallel in time \( T_{\text{parallel}} \), then we can also solve it sequentially in time \( T_{\text{sequential}} \leq c \cdot T_{\text{parallel}}^4 \). For computing \( n! \), the smallest possible time is \( n - 1 \): we need \( n - 1 \) multiplications. Thus,

\[
n - 1 \leq c \cdot T_{\text{parallel}}^4.
\]

Dividing both sides by \( c \), we get \( c^{-1} \cdot (n - 1) \leq T_{\text{parallel}}^4 \), hence

\[
T_{\text{parallel}} \geq C \cdot (n - 1)^{1/4} = C \cdot \sqrt[4]{n - 1}.
\]

This is faster than the sequential time \( n - 1 \), but much slower than the time \( \text{const} \cdot \log(n) \) that we would have if we ignored communication time.