Solution to Homework 32

Problem. If we take into account communication time, how fast can you compute $n!$ numbers in parallel?

Solution. According to Section 2 of the corresponding lecture, if we can solve the problem in parallel in time T_{parallel}, then we can also solve it sequentially in time $T_{\text{sequential}} \le c \cdot T_{\text{parallel}}^4$. For computing $n!$, the smallest possible time is $n-1$: we need $n-1$ multiplications. Thus,

$$n - 1 \le c \cdot T_{\text{parallel}}^4.$$

Dividing both sides by c, we get $c^{-1} \cdot (n-1) \le T_{\text{parallel}}^4$, hence

$$T_{\text{parallel}} \ge C \cdot (n-1)^{1/4} = C \cdot \sqrt[4]{n-1}.$$

This is faster than the sequential time $n-1$, but much slower than the time $\text{const} \cdot \log(n)$ that we would have if we ignored communication time.