Solution to Problem 6

Problem. Show that the following function \( f(a, b, c) \) is \( \mu \)-recursive:

- \( f(a, b, c) = a + b + c \) when each of the two inputs \( a \) and \( b \) is either equal to 0 or equal to 1, and
- \( f(a, b, c) \) is undefined for other triples \((a, b, c)\).

Possible solution. A natural idea is to take
\[
f(a, b, c) = \mu m.((a = 0 \lor a = 1) \land (b = 0 \lor b = 1) \land (m = a + b + c)).
\]

Another possible solution.
\[
f(a, b, c) = \mu m.((a = 0 \land b = 0 \land m = c) \lor (a = 0 \land b = 1 \land m = c + 1) \lor (a = 1 \land b = 0 \land m = c + 1) \lor (a = 1 \land b = 1 \land m = c + 2)).
\]