

Solution to Problem 6

Problem. Show that the following function $f(a, b, c)$ is μ -recursive:

- $f(a, b, c) = a + b + c$ when each of the two inputs a and b is either equal to 0 or equal to 1, and
- $f(a, b, c)$ is undefined for other triples (a, b, c) .

Possible solution. A natural idea is to take

$$f(a, b, c) = \mu m.((a = 0 \vee a = 1) \& (b = 0 \vee b = 1) \& (m = a + b + c)).$$

Another possible solution.

$$f(a, b, c) = \mu m.((a = 0 \& b = 0 \& m = c) \vee (a = 0 \& b = 1 \& m = c + 1) \vee (a = 1 \& b = 0 \& m = c + 1) \vee (a = 1 \& b = 1 \& m = c + 2)).$$