

## Solution to Problem 7

**Problem.** Prove that the following function is mu-recursive:

```
int j = 1;
while(!(a + j <= m))
  {j++;}
```

**Solution.** According to the general algorithm, first, we write a similar problem, but with a for-loop:

```
int j = 1;
for(i = 1; i <= b; i++)
  {j++;}
```

This program can be translated into primitive recursion as follows:

$$j(0) = 1;$$

$$j(n+1) = j(n) + 1.$$

A general primitive recursion defines a function  $h(n_1, \dots, n_k, m)$  of  $k+1$  variables. In our case, we have a function of 1 variable, so  $k+1 = 1$  and  $k = 0$ . For  $k = 0$ , the general primitive recursion has the form

$$j(0) = f;$$

$$j(n+1) = h(n, j(n)).$$

Here,  $f(0) = 0$  and  $h = \sigma \circ \pi_2^2$ , so

$$j(m) = PR(0, \sigma \circ \pi_2^2).$$

As the number of iterations, we take the smallest  $n$  for which  $a + j(n) \leq m$ , i.e.,  $\mu n.(a + j(n) \leq m)$ . Thus, the desired function has the form

$$j(\mu n.(a + j(n) \leq m)).$$