Solution to Problem 7

**Problem.** Prove that the following function is mu-recursive:

```c
int j = 1;
while(!(a + j <= m))
    {j++;}
```

**Solution.** According to the general algorithm, first, we write a similar problem, but with a for-loop:

```c
int j = 1;
for(i = 1; i <= b; i++)
    {j++;}
```

This program can be translated into primitive recursion as follows:

\[
\begin{align*}
    j(0) &= 1; \\
    j(n + 1) &= j(n) + 1.
\end{align*}
\]

A general primitive recursion defines a function \( h(n_1, \ldots, n_k, m) \) of \( k + 1 \) variables. In our case, we have a function of 1 variable, so \( k + 1 = 1 \) and \( k = 0 \). For \( k = 0 \), the general primitive recursion has the form

\[
\begin{align*}
    j(0) &= f; \\
    j(n + 1) &= h(n, j(n)).
\end{align*}
\]

Here, \( f(0) = 0 \) and \( h = \sigma \circ \pi_2^2 \), so

\[
j(m) = PR(0, \sigma \circ \pi_2^2).
\]

As the number of iterations, we take the smallest \( n \) for which \( a + j(n) \leq m \), i.e., \( \mu n. (a + j(n) \leq m) \). Thus, the desired function has the form

\[
j(\mu n. (a + j(n) \leq m)).
\]