

Solution to Problem 9

Problem. Design a Turing machine that computes a function $f(n)$ which is equal:

- to $n - 3$ when $n \geq 3$ and
- to 0 when $n = 0$, $n = 1$, or $n = 2$;

it is OK to assume that the input n is given in unary code.

Solution. The following rules take care of the case when $n \geq 3$:

- start, $- \rightarrow R$, moving
- moving, $1 \rightarrow R$
- moving, $- \rightarrow$ erasing1st
- erasing1st, $1 \rightarrow -, L$, erasing2nd
- erasing2nd, $1 \rightarrow -, L$, erasing3rd
- erasing3rd, $1 \rightarrow -, L$, back
- back, $1 \rightarrow L$
- back, $- \rightarrow$ halt

For $n = 0$, we get a situation when we are in the state erasing1st and we see blank $-$ since we are in the very first cell. To deal with this case, we need to add a rule

- erasing1st, $- \rightarrow$ halt

For $n = 1$, we encounter a similar situation when we are in the state erasing2nd and we see blank. So, we need to add one more rule:

- erasing2nd, $- \rightarrow$ halt

For $n = 2$, we encounter a similar situation when we are in the state erasing3rd and we see blank. So, we need to add one more rule:

- erasing3rd, $- \rightarrow$ halt