

Theory of Computation, Spring 2022, Test 3

Problem 1. Describe the two places where in the proof that propositional satisfiability is NP-hard we use the two main physical assumptions: that all the speeds are bounded by the speed of the light and that the volume of the sphere is proportional to the cube of its radius.

Problem 2. Use the general algorithm to translate the formula

$$(\neg a \vee b \vee \neg c \vee d) \& (a \vee b)$$

into 3-CNF.

Problem 3–4. Reduce the satisfiability problem for the formula

$$(\neg a \vee \neg b \vee c) \& (a \vee \neg b)$$

to:

- 3-coloring,
- clique,
- subset sum problem, and
- interval computations.

In all these reductions, explain what will correspond to $a = T$, $b = c = F$.

Problem 5. Show how to compute the maximum of 10 numbers in parallel if we have an unlimited number of processors and we can ignore communication time. Why do we need parallel processing in the first place? If we take communication time into account, how much time do we need to compute the maximum of n numbers? What is NC? Give an example of a P-complete problem.

Problem 6. What can you say about the Kolmogorov complexity of the following string: 010011000111... all the way to 2022 0s followed by 2022 1s.

Problem 7. Suppose that we have a probabilistic algorithm that gives a correct answer 9/10 of the time. How many times do we need to repeat this algorithm to reduce probability of error to at most 0.05%? Give an example of a probabilistic algorithm. Explain why we need probabilistic algorithms in the first place.

Problem 8. Use the variable-elimination algorithm for checking satisfiability of the following 2-SAT formula:

$$(a \vee \neg c) \& (\neg b \vee \neg c) \& (a \vee b) \& (\neg c \vee b) \& (\neg a \vee b).$$

Find all solutions.

Problem 9. How is “or”-operation represented in quantum computing? Provide a general formula and explain it on the example when one of the inputs is true, and another one is false.

Problem 10. Briefly describe what you have done for your project.