Solution to Homework 27

**Problem.** On the example of the formula \((\neg a \lor b \lor \neg c) \& (\neg a \lor \neg b)\), show how checking its satisfiability can be reduced to coloring a graph in 3 colors.

**Solution.** According to the general algorithm, first, we build a palette: three vertices T, and F, and U all connected to each other. Then, we:

- add two vertices \(a\) and \(\neg a\), and connect both of them to U and to each other;
- add two vertices \(b\) and \(\neg b\), and connect both of them to U and to each other;
- add two vertices \(c\) and \(\neg c\), and connect both of them to U and to each other.

For the 3-literal clause \(C_1 = \neg a \lor b \lor \neg c\), we:

- add a new vertex \(\neg a \lor b\),
- we connect this vertex to U,
- we add an or-gadget for \((\neg a \lor b) \lor \neg c\), i.e., we add new vertices \((\neg a \lor b)_1\) and \(\neg c_1\), connect both these two vertices to T and to each other, and connect \((\neg a \lor b)_1\) to \(\neg a \lor b\) and \(\neg c_1\) to \(\neg c\);
- we add vertices \(\neg a_1\) and \(b_1\) and connect them to the vertex \(\neg a \lor b\) and to each other, and
- we connect \(\neg a_1\) to \(\neg a\) and \(b_1\) to \(b\).

After that, for the 2-literal clause \(C_2 = \neg a \lor \neg b\), we add an or-gadget: namely,

- we add two new vertices \(\neg a_2\) and \(\neg b_2\),
- we connect both vertices \(\neg a_2\) and \(\neg b_2\) to T and to each other, and
- we connect \(\neg a\) to \(\neg a_2\) and \(\neg b\) to \(\neg b_2\).