Solution to Homework 32

Problem. If we take into account communication time, how fast can you compute
\[ 1 + 1/2 + \ldots + 1/n \]
in parallel?

Solution. According to Section 2 of the corresponding lecture, if we can solve the problem in parallel in time \( T_{\text{parallel}} \), then we can also solve it sequentially in time \( T_{\text{sequential}} \leq c \cdot T_{\text{parallel}}^4 \). For computing the above sum, the smallest possible time is \( 2n - 2 \): we need \( n - 1 \) divisions and \( n - 1 \) additions. Thus, \( 2n - 2 \leq c \cdot T_{\text{parallel}}^4 \). Dividing both sides by \( c \), we get \( c^{-1} \cdot (2n - 2) \leq T_{\text{parallel}}^4 \), hence
\[ T_{\text{parallel}} \geq C \cdot (2n - 2)^{1/4} = C \cdot \sqrt[4]{2n - 2}. \]

This is faster than the sequential time \( 2n - 2 \), but much slower than the time \( \text{const} \cdot \log(n) \) that we would have if we ignored communication time.