Solution to Problem 6

Problem. Show that the following function $f(a, b)$ is $\mu$-recursive:

- $f(a, b) = a \& b$ when each of the two inputs $a$ and $b$ is either equal to 0 or equal to 1, and
- $f(a, b)$ is undefined for other pairs $(a, b)$.

Possible solution. A natural idea is to take

$$f(a, b) = \mu m.((a = 0 \lor a = 1) \land (b = 0 \lor b = 1) \land (m = a \& b)).$$

Another possible solution.

$$f(a, b) = \mu m.((a = 0 \land b = 0 \land m = 0) \lor (a = 0 \land b = 1 \land m = 0) \lor (a = 1 \land b = 0 \land m = 0) \lor (a = 1 \land b = 1 \land m = 1)).$$