Solution to Problem 20

Problem. Suppose that $A, B$ are r.e. sets. If a number $n$ appears in the $A$-generating algorithm at moment 3, and the complement $\neg A$ is also r.e., when will the deciding algorithm tell us that $n$ is an element of the set $A$?

Solution. According to the lecture, we do the following:

1. First, we run the $A$-generating algorithm for 1 moment of time. This takes the 1st moment of time of the $A$-deciding algorithm.

2. Then, we run the $(\neg A)$-generating algorithm for 1 moment of time. This takes the 2nd moment of time of the $A$-deciding algorithm.

3. Then, we run the $A$-generating algorithm for 1 more moment of time. This takes the 3rd moment of time of the $A$-deciding algorithm.

4. Then, we run the $(\neg A)$-generating algorithm for 1 more moment of time. This takes the 4th moment of time of the $A$-deciding algorithm.

5. Then, we run the $A$-generating algorithm for 1 more moment of time. During this moment of time, the $A$-generating algorithm produces the desired number $n$ (and maybe some other numbers). This computation takes the 5th moment of time of the $A$-deciding algorithm.

So, the $A$-deciding algorithm will conclude that $n$ is in the set $A$ at moment 5.