

## Solution to Problem 22

**Problem.** Give:

- an example of computation time  $t_A(x)$  for which the algorithm is practically not feasible, but is feasible according to the existing definition, and
- an example of computation time  $t_A(x)$  for which the algorithm is practically feasible, but is not feasible according to the existing definition.

These examples must be different from the ones we had in class.

**Solution.**

First example:  $t_A(x) = 10^{2024}$ . This is a constant – so it is feasible in the sense of the formal definition. On the other hand, in class, we learned that:

- even if we have as many computational devices as physically possible – i.e., if every single elementary particle – and there are  $10^{90}$  of them – serves as a computational,
- and even if each of these computational devices performs one computational steps during each shortest possible periods of time – and there are about  $10^{40}$  of them during the lifetime of the Universe,

then overall, we can perform no more than  $10^{90} \cdot 10^{40} = 10^{130}$  computational steps, and  $10^{2024}$  is larger than  $10^{130}$ .

Second example:  $t_A(x) = \exp(10^{-2024} \cdot \text{len}(x))$ . This function is exponentially growing – thus, not feasible in the sense of the formal definition, since every exponential function grows faster than a polynomial.

However, in practice, the length of the input cannot be larger than the length that would get if we combine all the knowledge that we have in the world – which would be approximately  $\text{len}(x) = 10^{20}$  bits. Even for this huge number of bits, this algorithm would require

$$t_A(x) = \exp(10^{-2024} \cdot 10^{20}) = \exp(10^{-2004})$$

computational steps. Since  $10^{-2004}$  is smaller than 1 and  $\exp(x) = e^x$  is an increasing function, we conclude that

$$t_A(x) = \exp(10^{-2004}) \leq \exp(1) = 2.7128\dots,$$

i.e., this algorithm would require 1 or 2 steps, which is clearly feasible. If the input is shorter than  $10^{20}$  bits, we will need even fewer computational steps.