Solution to Problem 24

Problem. Use the general algorithm to come up with DNF form and CNF form of the formula $0.5 \cdot x_1 + 0.4 \cdot x_2 \geq 0.8 \cdot x_3$.

Solution. Let us describe the truth values of this formula F for all possible combinations of values x_1, x_2, and x_3.

x_1	x_2	x_3	F	$\neg F$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

The DNF form describes that the formula is true if we are in one the rows for which $F = 1$. So, the DNF form is as follows:

$$(\neg x_1 \mathbin{\&} \neg x_2 \mathbin{\&} \neg x_3) \vee (\neg x_1 \mathbin{\&} x_2 \mathbin{\&} \neg x_3) \vee (x_1 \mathbin{\&} \neg x_2 \mathbin{\&} \neg x_3) \vee$$

$$(x_1 \mathbin{\&} x_2 \mathbin{\&} \neg x_3) \vee (x_1 \mathbin{\&} x_2 \mathbin{\&} x_3).$$

To get the CNF form, we first need to write down the DNF form for the negation $\neg F$:

$$(\neg x_1 \mathbin{\&} \neg x_2 \mathbin{\&} x_3) \vee (\neg x_1 \mathbin{\&} x_2 \mathbin{\&} x_3) \vee (x_1 \mathbin{\&} \neg x_2 \mathbin{\&} x_3).$$

The CNF form is the negation of the DNF form for $\neg F$, obtained by using de Morgan laws:

$$(x_1 \vee x_2 \vee \neg x_3) \mathbin{\&} (x_1 \vee \neg x_2 \vee \neg x_3) \mathbin{\&} (\neg x_1 \vee x_2 \vee \neg x_3).$$