

## Solution to Homework 27

**Problem.** On the example of the formula  $(a \vee b \vee \neg c) \& (\neg a \vee \neg b)$ , show how checking its satisfiability can be reduced to coloring a graph in 3 colors.

**Solution.** According to the general algorithm, first, we build a palette: three vertices T, and F, and U all connected to each other. Then, we:

- add two vertices  $a$  and  $\neg a$ , and connect both of them to U and to each other;
- add two vertices  $b$  and  $\neg b$ , and connect both of them to U and to each other;
- add two vertices  $c$  and  $\neg c$ , and connect both of them to U and to each other.

For the 3-literal clause  $C_1 = a \vee b \vee \neg c$ , we:

- add a new vertex  $a \vee b$ ,
- we connect this vertex to U,
- we add an or-gadget for  $(a \vee b) \vee \neg c$ , i.e., we add new vertices  $(a \vee b)_1$  and  $\neg c_1$ , connect both these two vertices to T and to each other, and connect  $(a \vee b)_1$  to  $a \vee b$  and  $\neg c_1$  to  $\neg c$ ;
- we add vertices  $a_1$  and  $b_1$  and connect them to the vertex  $a \vee b$  and to each other, and
- we connect  $a_1$  to  $a$  and  $b_1$  to  $b$ .

After that, for the 2-literal clause  $C_2 = \neg a \vee \neg b$ , we add an or-gadget: namely,

- we add two new vertices  $\neg a_2$  and  $\neg b_2$ ,
- we connect both vertices  $\neg a_2$  and  $\neg b_2$  to T and to each other, and
- we connect  $\neg a$  to  $\neg a_2$  and  $\neg b$  to  $\neg b_2$ .