Solution to Homework 28

Problem. On the example of the formula $(a \vee b \vee \neg c) \, \& \, (\neg a \vee \neg b)$, show how checking its satisfiability can be reduced to an instance of the clique problem.

Solution. According to the general algorithm, since we have 2 clauses, we will be looking for cliques of size 2, i.e., edges.

To construct the corresponding graph, we do the following. First, we place two groups of vertices:

- vertices a_1, b_1, and $\neg c_1$ corresponding to the first clause; and

- vertices $\neg a_2$ and $\neg b_2$ corresponding to the second clause.

The following pairs of vertices are connected by edges:

- the vertex a_1 is connected to $\neg b_2$ (but *not* to $\neg a_2$);

- the vertex b_1 is connected to $\neg a_2$ (but *not* to $\neg b_2$); and

- the vertex $\neg c_1$ is connected to $\neg a_2$ and to $\neg b_2$.

These edges are the desired cliques of size 2.