

Solution to Homework 29

Problem. On the example of the formula $(a \vee b \vee \neg c) \& (\neg a \vee \neg b)$, show how checking its satisfiability can be reduced to an instance of the subset sum problem (i.e., the problem of exact change).

Solution. In the above 3-CNF formula, we have:

- three Boolean variables a , b , and c (so that $\ell = 3$), and
- $k = 2$ clauses $C_1 = a \vee b \vee \neg c$ and $C_2 = \neg a \vee \neg b$.

By applying the general algorithm, we get the following table:

	a	b	c	C_1	C_2
a	1	0	0	1	0
$\neg a$	1	0	0	0	1
b	0	1	0	1	0
$\neg b$	0	1	0	0	1
c	0	0	1	0	0
$\neg c$	0	0	1	1	0
C'_1	0	0	0	1	0
C''_1	0	0	0	1	0
C'_2	0	0	0	0	1
C''_2	0	0	0	0	1
S	1	1	1	3	3

So, we need to describe the number 11133 as the sum of a subset of the following values:

$$x_a = 10010, \quad x_{\neg a} = 10001, \quad x_b = 01010, \quad x_{\neg b} = 01001, \quad x_c = 00100, \quad x_{\neg c} = 00110,$$

$$x'_1 = 00010, \quad x''_1 = 00010, \quad x'_2 = 00001, \quad x''_2 = 00001.$$