Solution to Problem 31

**Problem.** Show how to compute the “or” of 10 boolean values in parallel if we have unlimited number of processors. How many processors do we need and how much time will the computation take? Why do we need parallel processing in the first place?

**Solution.** Suppose that we are given 10 boolean values $x_1, \ldots, x_{10}$, and we want to compute their conjunction (“or”). Then:

- at the first moment of time:
  - the first computer computes $x_1 \lor x_2$;
  - the second computer computes $x_3 \lor x_4$;
  - the third computer computes $x_5 \lor x_6$;
  - the forth computer computes $x_7 \lor x_8$;
  - the fifth computer computes $x_9 \lor x_{10}$;

- at the second moment of time:
  - the first computer computes $(x_1 \lor x_2) \lor (x_3 \lor x_4)$;
  - the second computer computes $(x_5 \lor x_6) \lor (x_7 \lor x_8)$.

- at the third moment of time, the first computer computes
  
  $x_1 \lor x_2 \lor x_3 \lor x_4 \lor x_5 \lor x_6 \lor x_7 \lor x_8$
  
  as
  
  $(x_1 \lor x_2 \lor x_3 \lor x_4) \lor (x_5 \lor x_6 \lor x_7 \lor x_8)$;

- at the fourth moment of time, the first computer computes the desired value
  
  $x_1 \lor x_2 \lor x_3 \lor x_4 \lor x_5 \lor x_6 \lor x_7 \lor x_8 \lor x_9 \lor x_{10}$
  
  as
  
  $(x_1 \lor x_2 \lor x_3 \lor x_4 \lor x_5 \lor x_6 \lor x_7 \lor x_8) \lor (x_9 \lor x_{10})$.

These computations require 5 computers and 4 moments of time.

In general, parallel computations are needed to speed up computations. Without parallelism, we would need 10 moments of time to compute the desired product.