Solution to Homework 32

Problem. If we take into account communication time, how fast can you compute

$$1 + \frac{1}{2^2} + \ldots + \frac{1}{n^2}$$

in parallel?

Solution. According to Section 2 of the corresponding lecture, if we can solve the problem in parallel in time $T_{\rm parallel}$, then we can also solve it sequentially in time $T_{\rm sequential} \le c \cdot T^4_{\rm parallel}$. For computing the above sum, the smallest possible time is $3n-3$: we need $n-1$ multiplications to compute the squares, $n-1$ divisions, and $n-1$ additions. Thus, $3n-3 \le c \cdot T^4_{\rm parallel}$. Dividing both sides by c, we get $c^{-1} \cdot (3n-3) \le T^4_{\rm parallel}$, hence

$$T_{\rm parallel} \ge C \cdot (3n-3)^{1/4} = C \cdot \sqrt[4]{3n-3}.$$

This is faster than the sequential time $3n-3$, but much slower than the time $\mathrm{const} \cdot \log(n)$ that we would have if we ignored communication time.