

Solution to Problem 6

Problem. Show that the following function $f(a, b)$ is μ -recursive:

- $f(a, b) = a \& \neg b$ when each of the two inputs a and b is either equal to 0 or equal to 1, and
- $f(a, b)$ is undefined for other pairs (a, b) .

Possible solution. A natural idea is to take

$$f(a, b) = \mu m.((a = 0 \vee a = 1) \& (b = 0 \vee b = 1) \& (m = a \& \neg b)).$$

Another possible solution.

$$f(a, b) = \mu m.((a = 0 \& b = 0 \& m = 0) \vee (a = 0 \& b = 1 \& m = 0) \vee (a = 1 \& b = 0 \& m = 1) \vee (a = 1 \& b = 1 \& m = 0)).$$