Solution to Problem 6

Problem. Show that the following function \( f(a, b) \) is \( \mu \)-recursive:

- \( f(a, b) = a \& \neg b \) when each of the two inputs \( a \) and \( b \) is either equal to 0 or equal to 1, and
- \( f(a, b) \) is undefined for other pairs \( (a, b) \).

Possible solution. A natural idea is to take

\[
f(a, b) = \mu m.((a = 0 \lor a = 1) \& (b = 0 \lor b = 1) \& (m = a \& \neg b)).
\]

Another possible solution.

\[
f(a, b) = \mu m.((a = 0 \& b = 0 \& m = 0) \lor (a = 0 \& b = 1 \& m = 0) \lor (a = 1 \& b = 0 \& m = 1) \lor (a = 1 \& b = 1 \& m = 0)).
\]