Solution to Problem 7

Problem. Prove that the following function is mu-recursive:

```
int j = a;
while(!(j * j <= b))
  {j++;}
```

Solution. According to the general algorithm, first, we write a similar problem, but with a for-loop:

```
int j = a;
for(i = 1; i <= m; i++)
  {j++;}
```

This program can be translated into primitive recursion as follows:

$$j(a, 0) = a;$$

$$j(a, m + 1) = j(a, m) + 1.$$

A general primitive recursion defines a function $h(n_1, \ldots, n_k, m)$ of $k + 1$ variables. In our case, we have a function of 2 variables, so $k + 1 = 2$ and $k = 1$. For $k = 1$, the general primitive recursion has the form

$$j(a, 0) = f(a);$$

$$j(a, m + 1) = g(a, m, j(a, m)).$$

Here, $f(a) = a = \pi_1^1$ and $g = \sigma \circ \pi_3^3$, so

$$j(m) = PR(\pi_1^1, \sigma \circ \pi_3^3).$$

As the number of iterations, we take the smallest m for which $!(j(a, m) * j(a, m) \le b)$, i.e., $\mu m.!(j(a, m) * j(a, m) \le b)$. Thus, the desired function has the form

$$j(a, \mu m.!(j(a, m) * j(a, m) \le b)).$$