Theory of Computation, Spring 2024
Test 3

Problem 1. Describe how, in the derivation of the lower bound for parallel computation time, we use the two main physical assumptions: that the volume of the sphere is proportional to the cube of its radius and that all the speeds are bounded by the speed of the light.

Problem 2. Use the general algorithm to translate the formula

$$(\neg p \vee q \vee r \vee s) \,\&\, (\neg s \vee t)$$

into 3-CNF.

Problem 3–4. Reduce the satisfiability problem for the formula

$$(\neg a \vee \neg b \vee \neg c) \,\&\, (a \vee \neg b)$$

to:

- 3-coloring,

- clique,

- subset sum problem, and

- interval computations.

In all these reductions, explain what will correspond to $a = T$, $b = F$, and $c = T$.

Problem 5. Show how to compute the "or" of 12 Boolean values in parallel if we have an unlimited number of processors and we can ignore communication time. Why do we need parallel processing in the first place? If we take communication time into account, how much time do we need to compute the product of n numbers? What is NC? Give an example of a P-complete problem.

Problem 6. What can you say about the Kolmogorov complexity of the following string: 101101... in which 101 is repeated 17,000 times.

<center>Turn over, please!</center>

Problem 7. Suppose that we have a probabilistic algorithm that gives a correct answer half of the time. How many times do we need to repeat this algorithm to reduce probability of error to at most 10%? Give an example of a probabilistic algorithm. Explain why we need probabilistic algorithms in the first place.

Problem 8. Use the variable-elimination algorithm for checking satisfiability of the following 2-SAT formula:

$$(\neg a \vee c) \mathbin{\&} (\neg b \vee \neg c) \mathbin{\&} (a \vee b) \mathbin{\&} (\neg c \vee b).$$

Find all solutions.

Problem 9. How is the negation operation $f(x_1) = \neg x_1$ – which is true when x_1 is false and false when x_1 is true – represented in quantum computing? Provide a general formula and explain it on the example when x_1 is true and the auxiliary variable is false.

Problem 10. Briefly describe what you have done for your project.