

## Solution to Problem 6

**Problem.** Show that the following function  $f(a, b)$  is  $\mu$ -recursive:

- $f(a, b) = \neg a \ \& \ \neg b$  when each of the two inputs  $a$  and  $b$  is either equal to 0 or equal to 1, and
- $f(a, b)$  is undefined for other pairs  $(a, b)$ .

**Possible solution.** A natural idea is to take

$$f(a, b) = \mu m.((a = 0 \vee a = 1) \ \& \ (b = 0 \vee b = 1) \ \& \ (m = \neg a \ \& \ \neg b)).$$

**Another possible solution.**

$$f(a, b) = \mu m.((a = 0 \ \& \ b = 0 \ \& \ m = 1) \vee (a = 0 \ \& \ b = 1 \ \& \ m = 0) \vee \\ (a = 1 \ \& \ b = 0 \ \& \ m = 0) \vee (a = 1 \ \& \ b = 1 \ \& \ m = 0)).$$