Solution to Problem 10

Problem. In class, we designed a Turing machine for computing π^2_1. Use this design as a sample to design a Turing machine for computing π^4_1 for tuples of unary numbers. Trace, step-by-step, on an example, how your Turing machine works. For example, you can take as input the tuple $(2, 1, 1, 3)$.

Reminder: The Turing machine for computing π^2_1 for tuples of unary numbers is based on the following idea:

- we go step-by-step until we find the first blank space,
- when we see blank, this means that we are outside the 1st number, and we are going inside the second number, so we continue going right,
- when we see blank again, this means that we reached the end of the second number, so we go back and erase 1s one by one until we reach the blank space separating the 2nd number from the 1st one,
- after that, we stop erasing and simply go back.

This Turing machine has the following rules:

- start, $-$ $\to$ R, in1st
- in1st, 1 $\to$ R
- in1st, $-$ $\to$ R, in2nd
- in2nd, 1 $\to$ R
- in2nd, $-$ $\to$ L, erasing
- erasing, 1 $\to$ $-$, L
- erasing, $-$ $\to$ L, back
- back, 1 $\to$ L
- back, $-$ $\to$ halt.

Solution. Now, we need to erase the second, the third, and the fourth numbers. So, when we reach the end of the second number, we do not yet start erasing: first, we do through the 3rd and the 4th numbers and only start erasing when we reach the end of the 4th number:

- start, – → R, in1st
- in1st, 1 → R
- in1st, – → R, in2nd
- in2nd, 1 → R
- in2nd, – → R, in3rd
- in3rd, 1 → R
- in3rd, – → R, in4th
- in4th, 1 → R
- in4th, – → L, erasing4th
- erasing4th, 1 → –, L
- erasing4th, – → L, erasing3rd
- erasing3rd, 1 → –, L
- erasing3rd, – → L, erasing2nd
- erasing2nd, 1 → –, L
- erasing2nd, – → L, back
- back, 1 → L
- back, – → halt.

Let us trace this idea on the example of the tuple $(2, 1, 1, 3)$:

$\underline{-}$	1	1	−	1	−	1	−	1	1	1	−	...	start
−	$\underline{1}$	1	−	1	−	1	−	1	1	1	−	...	in1st
−	1	$\underline{1}$	−	1	−	1	−	1	1	1	−	...	in1st
−	1	1	$\underline{-}$	1	−	1	−	1	1	1	−	...	in1st
−	1	1	−	$\underline{1}$	−	1	−	1	1	1	−	...	in2nd
−	1	1	−	1	$\underline{-}$	1	−	1	1	1	−	...	in2nd
−	1	1	−	1	−	$\underline{1}$	−	1	1	1	−	...	in3rd
−	1	1	−	1	−	1	$\underline{-}$	1	1	1	−	...	in3rd
−	1	1	−	1	−	1	−	$\underline{1}$	1	1	−	...	in4th
−	1	1	−	1	−	1	−	1	$\underline{1}$	1	−	...	in4th

$-$	1	1	$-$	1	$-$	1	$-$	1	1	$\underline{1}$	$-$	…	in4th
$-$	1	1	$-$	1	$-$	1	$-$	1	1	1	$\underline{-}$	…	in4th
$-$	1	1	$-$	1	$-$	1	$-$	1	1	$\underline{1}$	$-$	…	erasing4th
$-$	1	1	$-$	1	$-$	1	$-$	1	$\underline{1}$	$-$	$-$	…	erasing4th
$-$	1	1	$-$	1	$-$	1	$-$	$\underline{1}$	$-$	$-$	$-$	…	erasing4th
$-$	1	1	$-$	1	$-$	1	$\underline{-}$	$-$	$-$	$-$	$-$	…	erasing4th
$-$	1	1	$-$	1	$-$	$\underline{1}$	$-$	$-$	$-$	$-$	$-$	…	erasing3rd
$-$	1	1	$-$	1	$\underline{-}$	$-$	$-$	$-$	$-$	$-$	$-$	…	erasing3rd
$-$	1	1	$-$	$\underline{1}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	…	erasing2nd
$-$	1	1	$\underline{-}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	…	erasing2nd
$-$	1	$\underline{1}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	…	back
$-$	$\underline{1}$	1	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	…	back
$\underline{-}$	1	1	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	…	back
$\underline{-}$	1	1	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	…	halt