Solution to Homework 27

Problem. On the example of the formula $(\neg a \vee b \vee \neg c)\,\&\,(a \vee \neg b)$, show how checking its satisfiability can be reduced to an instance of the subset sum problem (i.e., the problem of exact change).

Solution. In the above 3-CNF formula, we have:

- three Boolean variables a, b, and c (so that $\ell = 3$), and
- $k = 2$ clauses $C_1 = \neg a \vee b \vee \neg c$ and $C_2 = a \vee \neg b$.

By applying the general algorithm, we get the following table:

	a	b	c	C_1	C_2
a	1	0	0	0	1
$\neg a$	1	0	0	1	0
b	0	1	0	1	0
$\neg b$	0	1	0	0	1
c	0	0	1	0	0
$\neg c$	0	0	1	1	0
C_1'	0	0	0	1	0
C_1''	0	0	0	1	0
C_2'	0	0	0	0	1
C_2''	0	0	0	0	1
S	1	1	1	3	3

So, we need to describe the number 11133 as the sum of a subset of the following values:

$$x_a = 10001,\ \ x_{\neg a} = 10010,\ \ x_b = 01010,\ \ x_{\neg b} = 01001, x_c = 00100, x_{\neg c} = 00110,$$

$$x_1' = 00010,\ \ x_1'' = 00010,\ \ x_2' = 00001,\ \ x_2'' = 00001.$$