Solution to Homework 30

Problem. If we take into account communication time, how fast can you compute

$$1 + \frac{1}{2} + \ldots + \frac{1}{n}$$

in parallel?

Solution. According to Section 2 of the corresponding lecture, if we can solve the problem in parallel in time T_{parallel}, then we can also solve it sequentially in time $T_{\text{sequential}} \le c \cdot T^4_{\text{parallel}}$. For computing the above sum, the smallest possible time is $2n-2$: we need $n-1$ divisions and $n-1$ additions. Thus, $2n-2 \le c \cdot T^4_{\text{parallel}}$. Dividing both sides by c, we get $c^{-1} \cdot (2n-2) \le T^4_{\text{parallel}}$, hence

$$T_{\text{parallel}} \ge C \cdot (2n-2)^{1/4} = C \cdot \sqrt[4]{2n-2}.$$

This is faster than the sequential time $2n-2$, but much slower than the time $\text{const} \cdot \log(n)$ that we would have if we ignored communication time.