Theory of Computation, Test 3, Spring 2025

Problem 1. Explain where and how, in proof that satisfiability is NP-hard, we use the two physical assumptions: that all speeds are bounded by the speed of light, and that the volume of a sphere is proportional to the cube of its radius.

Problem 2. Use the general algorithm to translate the formula

$$(p \vee \neg q \vee r \vee s) \,\&\, (\neg s \vee t)$$

into 3-CNF.

Problem 3–4. Reduce the satisfiability problem for the formula

$$(a \vee b \vee \neg c) \,\&\, (a \vee \neg b)$$

to:

- 3-coloring,
- clique,
- subset sum problem, and
- interval computations.

In all these reductions, explain what will correspond to $a = T$, $b = F$, and $c = T$.

Problem 5. Show how to compute the "and" of 12 Boolean values in parallel if we have an unlimited number of processors and we can ignore communication time. Why do we need parallel processing in the first place? If we take communication time into account, how much time do we need to compute the "and" of n values? What is NC? Give an example of a P-complete problem.

Problem 6. What can you say about the Kolmogorov complexity of the following string: 110110... in which 110 is repeated 7,000 times.

Problem 7. Suppose that we have a probabilistic algorithm that gives a correct answer half of the time. How many times do we need to repeat this algorithm to reduce probability of error to at most 5%? Give an example of a probabilistic algorithm. Explain why we need probabilistic algorithms in the first place.

Turn over, please

Problem 8. Use the variable-elimination algorithm for checking satisfiability of the following 2-SAT formula:

$$(a \vee \neg c) \,\&\, (b \vee c) \,\&\, (\neg a \vee \neg b) \,\&\, (c \vee \neg b).$$

Find all solutions.

Problem 9. How is the "and" operation $f(x_1, x_2) = x_1 \,\&\, x_2$ represented in quantum computing? Provide a general formula and explain it on the example when x_1 is true and both x_2 and the auxiliary variable are false.

Problem 10. Why do we need to study recursively enumerable (r.e.) sets? Is the intersection of three r.e. sets still r.e.? If yes, prove it, if no, provide a counterexample.