Solution to Problem 17

Problem. Give:

- an example of computation time $t_A(x)$ for which the algorithm is practically not feasible, but is feasible according to the existing definition, and
- an example of computation time $t_A(x)$ for which the algorithm is practically feasible, but is not feasible according to the existing definition.

These examples must be different from the ones we had in class.

Solution.

First example: $t_A(x) = 10^{2026}$. This is a constant – so it is feasible in the sense of the formal definition. On the other hand, in class, we learned that:

- even if we have as many computational devices as physically possible – i.e., if every single elementary particle – and there are 10^{90} of them – serves as a computational,
- and even if each of these computational devices performs one computational steps during each shortest possible periods of time – and there are about 10^{40} of them during the lifetime of the Universe,

then overall, we can perform no more than $10^{90} \cdot 10^{40} = 10^{130}$ computational steps, and 10^{2026} is larger than 10^{130}.

Second example: $t_A(x) = \exp\left(10^{-2026} \cdot \text{len}(x)\right)$. This function is exponentially growing – thus, not feasible in the sense of the formal definition, since every exponential function grows faster than a polynomial.

However, in practice, the length of the input cannot be larger than the length that would get if we combine all the knowledge that we have in the world – which would be approximately $\text{len}(x) = 10^{20}$ bits. Even for this huge number of bits, this algorithm would require

$$t_A(x) = \exp\left(10^{-2026} \cdot 10^{20}\right) = \exp\left(10^{-2006}\right)$$

computational steps. Since 10^{-2006} is smaller than 1 and $\exp(x) = e^x$ is an increasing function, we conclude that

$$t_A(x) = \exp\left(10^{-2006}\right) \le \exp(1) = 2.7128\ldots,$$

i.e., this algorithm would require 1 or 2 steps, which is clearly feasible. If the input is shorter than 10^{20} bits, we will need even fewer computational steps.